Robustness and informativeness of systemic risk measures[1]

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Abstract

Recent literature has proposed new methods for measuring the systemic risk of financial institutions based on observed stock returns. In this paper we examine the reliability and robustness of such risk measures, focusing on CoVaR, marginal expected shortfall, and option-based tail risk estimates. We show that CoVaR exhibits undesired characteristics in the way it responds to idiosyncratic risk. In the presence of contagion, the risk measures provide conflicting signals on the systemic risk of infectious and infected banks. Finally, we explore how limited data availability typical of practical applications may limit the measures' performance. We generate systemic tail risk through positions in standard index options and describe situations in which systemic risk is misestimated by the three measures. The observations raise doubts about the informativeness of the proposed measures. In particular, a direct application to regulatory capital surcharges for systemic risk could create wrong incentives for banks.

Keywords: Systemic Risk; CoVaR; Marginal Expected Shortfall; Tail Risk

JEL-Classification: G21, G28

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1 Introduction

A key goal of financial regulation is to avoid a breakdown of the financial system, or at least keep the probability of such an event at acceptable levels. A breakdown is imminent if many relevant financial institutions are simultaneously put under stress. How much an individual bank contributes to system risk can depend on several factors, notably the size of the bank, its sensitivity to shocks, and the magnitude of spillovers to other banks. While this description of systemic risk factors may appear too broad, narrower definitions run the risk of missing important aspects. Contagious losses arising from the interconnectedness of banks as well as other externalities may exacerbate or even create a crisis, suggesting that an analysis of systemic risk should focus on interactions within the system. However, such interactions are not a necessary condition for a financial crisis. A severe shock that affects the entire economy, such as a large drop in housing prices, can be sufficient to create jeopardizing system-wide losses.

Though the importance of systemic risk has been discussed for many years,[2] bank regulation has been based mainly on stand-alone measures of an institution's risk. The recent financial crisis, however, has challenged trust in the underlying logic that the stability of a system can be adequately controlled if the stability of the system components is controlled individually. Reforms that are meant to overcome the shortcomings of traditional microprudential regulation are under discussion (see Hanson, Kashyap, and Stein, 2011). An important element of this is to make supervisory intensity and capital requirements dependent on a bank's systemic risk contribution.

As part of this discussion, a number of papers have proposed new empirical concepts for measuring systemic risk. Acharya, Pedersen, Philippon, and Richardson (2010) suggest examining marginal expected shortfall, which they measure through an institution's average equity return on days in which the market return is below its 5% quantile. Adrian and Brunnermeier (2011) advocate the CoVaR measure. This is the value at risk of the system conditional on an institution being in distress, or, alternatively, the value at risk of an institution conditional on the system being in distress. Knaup and Wagner (2012) recommend employing information from index options. Their measure of systemic risk is the sensitivity of an institution's equity returns to out-of-the money index put options.

[2] See De Bandt and Hartmann (2002) for an early survey.

The purpose of the present paper is to investigate the reliability of such return-based measures of systemic risk. We start by examining a linear market model framework to explore whether the measures adequately respond to differences in systematic and idiosyncratic risk. While the measures' sensitivity to systematic beta risk meets the expectation, the CoVaR response to idiosyncratic risk is ambiguous. In many situations, the use of CoVaR could create incentives for banks to increase their idiosyncratic risk in order to lower their estimated systemic risk.

Next, we examine a contagion framework in which negative shocks to one bank spill over to other banks. Depending on the parameterization, CoVaR measures can assign a higher systemic risk to the infected banks, while – in the situations analyzed here – marginal expected shortfall and option-based measures tend to assign the highest systemic risk to the infectious bank.

The problems described so far are conceptual and not due to data problems. In a next step, we explore how limited data availability typical of practical applications may limit the measures' performance. The events that regulators are concerned about are so extreme that it is quite likely that the data used for estimation do not contain such an event. Extant papers acknowledge this problem but argue that it can be contained if less extreme events are sufficiently informative about crisis events, if the estimation sample is sufficiently large, or if market expectations about crisis events are used.

Based on simulations, we arrive at a more pessimistic view. We generate systemic tail risk through positions in standard index options and describe several situations in which systemic risk is consistently misestimated. For example, protective put strategies that are immune to extreme shocks can be judged to have high systemic risk because estimation methods rely on less extreme return realizations in which option premia depress returns relative to unprotected institutions. On the other hand, tail risk can be masked by buying protection against less extreme events. None of the methods studied in the paper appears immune.

In our simulations, we use return profiles that are generated through index options. Sometimes the profiles involve relatively large option positions. This could be taken to question the relevance of our results because financial institutions may not be able to build up such large positions, or hide them if the return-based analysis is complemented by a holdings-based one. However, credit risk exposures also lead to non-linearities. From structural models (Merton, 1974) it is evident that a corporate loan or a credit default swap (CDS) includes an out-of-the-money put option on the borrower's assets. Recently, Carr and Wu (2011) have shown that there is a close relationship between CDS spreads and a spread between two deep out-of-the money puts on the borrower's equity.

Though we focus on the marginal expected shortfall (MES), CoVaR, and option sensitivity measures, these measures do not complete the list (for an overview, cf. Bisias et al. (2012)). Hartmann, Straetmans and de Vries (2006) and De Jonghe (2009) examine co-crash probabilities, a measure which is similar to marginal expected shortfall and CoVaR. Hautsch, Schaumburg, and Schienle (2011) propose the *systemic risk beta* which is conceptually closely related to the CoVaR. Brownlees and Engle (2011) and Acharya, Engle, and Richardson (2012) refine the MES concept by loss coverage through bank capital. Billio, Getmansky, Lo and Pelizzon (2010) use time series analysis to study interrelatedness. They examine autocorrelation, time variation in commonality, regime shifts and Granger causality. Since we examine processes with constant parameters combined with the assumption of efficient market prices, their methods would not help to discover systemic risk in our setup, which is why we cannot derive statements on their informativeness. In our somewhat idealized setup, which we choose in order to focus on conceptual issues, there is thus also no need to capture time-variation in risk, an empirical issue that is, for example, addressed in the work of Hautsch, Schaumburg and Schienle (2011) and Brownlees and Engle (2011).

In a set of papers, risk assessments are based on banks' default probabilities (Bartram, Brown and Hund, 2007; Huang, Zhou and Zhu, 2009; Segoviano and Goodhart, 2009). Another branch of the literature employs a holdings-based analysis of credit exposures, e.g. Upper and Worms (2004), Elsinger, Lehar, and Summer (2006), Martínez-Jaramillo, Pérez, Avila, and López (2010), Memmel and Sachs (2011), or Gauthier, Lehar and Souissi (2012). These studies are often based on bilateral exposures between banks (including repos and counterparty risk from OTC trading). Such detailed information allows direct modeling of the network externalities of a bank's default. For example, the *contagion index* proposed by Cont, Moussa, and Santos (2012) is the expected loss of other banks, conditional on the default of a certain bank and macroeconomic stress. Webber and Willison (2011) also use bilateral exposures and derive implicit measures. They optimize the capital allocation among banks while keeping the VaR of aggregate losses in the system under some limit. The resulting capital allocation, relative to the one without interbank linkages, can be interpreted as a systemic risk measure.

As practical calculations of such network based measures still require many strong assumptions, for example, on the clearing mechanism for interbank debt or the economic conditions under which an initial default occurs, there are attempts to link these measures to simpler ones from accounting (size, total interbank lending and borrowing, funding) or network theory. Evidence on their performance is mixed. While Gauthier, Gravelle, Liu, and Souissi (2011) find that bank size alone is not an appropriate proxy for systemic importance, Drehmann and Tarashev (2001) arrive at the opposite conclusion, at least for their preferred measure, the Shapley value based on expected shortfall. Theoretical results supporting the use of the Shapley value for attributing risk are presented in Tarashev, Borio, and Tsatsaronis (2010). Puzanova and Düllmann (2013), who use the MES to measure risk contributions, agree with Gauthier et al (2011), similar to Zhou (2010), who uses the probability based concept of Segoviano and Goodhart (2009). López-Espinosa et al. (2012) find that, among large international banks, short-term wholesale funding is the key driver of ΔCoVaR, while size appears to be less important.

Compared to the holdings-based approaches, which usually require data on bilateral interbank exposures or, at least, estimates of them, the equity-return based measures studied in this paper have the advantage of being based on readily observable prices.

Regulators have agreed on assessing systemic relevance with an indicator system that does not factor in methods from the above literature, except for the fact that a number of the accounting figures used as proxy measures of systemic importance show up as relevant bank-specific factors in the Basel documents on the identification of systemically important banks (Basel Committee on Banking Supervision, 2011a and 2012). However, the regulators also stress that the measurement of systemic risk is at "an early stage of development" (Basel Committee on Banking Supervision, 2011b, p. 2).

To our knowledge, there is only one study which examines the informativeness of market-based measures in a way that is similar to ours. Benoit, Colletaz, Hurlin, and Perignon (2012) theoretically analyze systemic risk measures in the model framework of Brownlees and Engle (2011) and find that MES and ΔCoVaR hardly provide any information in addition to that captured by market betas and volatilities. However, the result crucially depends on model linearity, while we show that nonlinearities can be significant both in the form of infection mechanisms and of nonlinear factor dependencies. Benoit et al. (2012) also perform an empirical analysis of US banks' daily stock returns. They find a strong relationship between a bank's MES and its market beta, from which one could conclude that the MES can properly be proxied by the beta. However, the authors compare *average* measures over a long period of ten years of daily returns. Whether a similar relationship would hold for the *current* estimates of MES and beta is unclear. Furthermore, the results are based on the commonly used 5% quantile level for the MES. Our analysis shows that the 5% level, which is a concession to data availability rather than a conceptually motivated choice, can lead to misleading systemic risk rankings for more extreme levels that regulators are actually interested in.

The remainder of the paper is structured as follows. In Section 2, we introduce the systemic risk measures studied in this paper. Section 3 discusses possible problems in a linear return setting, while Section 4 introduces contagion. In Section 5, we examine the ability of the risk measures to identify systemic tail risk in a setting that is typical of practical applications. Section 6 concludes.

2 Systemic risk measures studied in this paper

Marginal expected shortfall

The marginal expected shortfall (MES) put forward by Acharya et al. (2010) is defined as

$$MES_i = E\left(R_i \middle| R_m < Q_m^{\alpha}\right),$$

where R_i denotes the net equity return of institution i, R_m is the market return, and Q_m^{α} is the quantile of the market returns on level α. We follow Acharya et al. (2010) and examine daily returns with a confidence level α of 5%.

ΔCoVaR

Adrian and Brunnermeier suggest measures based on what they call CoVaR, which is implicitly defined through

$$\Pr\left(X^j \leq CoVaR_{\alpha}^{j|C(X^i)} \middle| C\left(X^i\right)\right) = \alpha$$

CoVaR therefore is the value at risk (VaR) of object j conditional on event C happening to object i. Taking the event to be that i is at its VaR level, they suggest to examine

$$\Delta CoVaR_{\alpha}^{j,i} = CoVaR_{\alpha}^{j|X^i=VaR_{\alpha}^i} - CoVaR_{\alpha}^{j|X^i=Median^i}.$$

$\Delta CoVaR_{\alpha}^{j,i}$ therefore measures the change in the α-VaR of j conditional on i moving from its median state to its own α-VaR. Adrian and Brunnermeier (2011) mostly examine the case in which j is given by the overall system, i.e. a market index or a collection of banks, and i

is an individual institution. However, they also consider the opposite direction in what they call *exposure CoVaR*, which is defined through

$$\Delta CoVaR_{\alpha}^{j,system} = CoVaR_{\alpha}^{j|X^{systemi}=VaR_{\alpha}^{system}} - CoVaR_{\alpha}^{j|X^{system}=Median^{system}}$$

$\Delta CoVaR_{\alpha}^{j,system}$ is the change in the VaR of portfolio j conditional on the system moving into distress. $\Delta CoVaR_{\alpha}^{j,system}$ is more akin to MES_i than $\Delta CoVaR_{\alpha}^{system,j}$.

When we examine CoVaR's expected performance in practical applications, we estimate it with a quantile regression over 25 years of weekly data, choosing a confidence level α of 1%. This corresponds to the empirical application in Adrian and Brunnermeier (2011). In the conceptual analysis of Sections 3 and 4, we abstract from estimation problems by deriving results through closed-form expression, or Monte Carlo simulations with a large number of observations.

Adrian and Brunnermeier (2011) suggest that, in the presence of time-varying risk, the precision of CoVaR estimates can be improved by conditioning the return-based estimates on current fundamental information. As we do not introduce time variation in risk parameters, the unconditional return-based estimates are optimal, which is why we do not model the conditional distribution. Another difference from the empirical approach of Adrian and Brunnermeier (2011) is that we examine equity returns rather than asset returns. This is done for the sake of exposition, as the other two measures examined in the paper are based on equity returns. While the choice of equity rather than asset returns can have an effect in practical applications, it does not affect the general results here. We model our equity returns as being normally distributed; this is also the standard assumption for asset returns in Merton (1974) type models. Thus, we could classify the returns in the CoVaR analysis as 'asset returns' and would still be in line with assumptions commonly made in the literature.

Tail risk gammas

Knaup and Wagner (2012) advocate the inclusion of forward-looking information available through market prices of out-of-the-money index put options. The sensitivity of an institution's equity return to changes in put option prices is estimated through the following linear regression:

$$R_{it} = a + bR_{mt} - \gamma \frac{p_t - p_{t-1}}{p_{t-1}} + u_t$$

where p_t denotes the option price of a put on the market index.[3]

The higher the estimated gamma, the higher the estimated systemic risk, conditional on the standard beta measure of systematic risk. In their empirical application, Knaup and Wagner study daily equity returns. They use put options with a maturity between three and six months; the average strike price is 67% of the index value. In line with their choice, we will use put options with a maturity of four months and a strike equal to 70% of the index value.

3 Systemic risk measures in the linear case

In this section, we use a linear return framework to examine whether the suggested measures for systemic risk fulfill elementary requirements with respect to a bank's choice of systematic and idiosyncratic risk.

Consider a banking system consisting of N banks. In the first case that we examine, R_i, the equity return of bank i, is determined by the exposure to a common risk factor F and idiosyncratic risk ε_i. Both factor returns and idiosyncratic components are assumed to be independent normal random variates. Let us further assume that the N banks do not differ in their market capitalization. The value-weighted index of bank returns is therefore equal to the simple average of the returns. We follow Adrian and Brunnermeier (2011) and take this system return to be the one that takes the role of a general market index, including the MES and tail risk gamma analysis.

The system and its components are described through the following equations:

$$\begin{gathered} R_i = \beta_i F + \varepsilon_i, \quad R_S = \frac{1}{N}\sum_{i=1}^{N} R_i \\ \text{with} \quad F \sim N\left(\mu, \sigma^2(F)\right), \quad \varepsilon_i \sim N\left(0, \sigma^2(\varepsilon_i)\right), \end{gathered} \tag{1}$$

where F and all ε_i are independent, β_i denotes the exposure to the common factor, and R_S is the return on the banking system index. To calculate measures of systemic risk, we need to specify conditional distributions. Due to the linearity of the system and the normality of the random variables, we can approach the problem in a linear regression framework.

[3] As derived by Knaup and Wagner (2012), the denominator of the put variable is $p_{t-1} + strike$ rather than p_{t-1}. Since we will use constant volatilities and constant strike prices, the definition of the denominator would not change the results. Our gamma estimates are perfectly linearly related to the gammas that would be obtained with the denominator of Knaup and Wagner (2012).

We start with an analysis of CoVaR measures. When we condition R_S on R_i, we study an orthogonal representation

$$R_S = c_i + d_i R_i + v_i,$$

and obtain:

$$\begin{aligned} d_i &= \frac{\operatorname{cov}(R_S, R_i)}{\sigma^2(R_i)} = \frac{\frac{1}{N}\sigma^2(R_i) + \frac{1}{N}\beta_i \sum_{j\neq i}\beta_j \sigma^2(F)}{\sigma^2(R_i)} = \frac{1}{N}\left(1 + \beta_i \sum_{j\neq i}\beta_j \frac{\sigma^2(F)}{\sigma^2(R_i)}\right) \\ c_i &= E(R_S) - d_i E(R_i) \\ \sigma^2(v_i) &= \sigma^2(R_S) - d_i^2 \sigma^2(R_i) \end{aligned} \tag{2}$$

When we use ΔCoVaR to study the extent to which the system is affected by bank i, we obtain:

$$\begin{aligned} \Delta CoVaR_q^{S,i} &= \left[c_i + d_i \Phi^{-1}(q)\sigma(R_i) + \Phi^{-1}(q)\sigma(v_i)\right] \\ &\quad - \left[c_i + d_i \Phi^{-1}(0.5)\sigma(R_i) + \Phi^{-1}(q)\sigma(v_i)\right] \\ &= d_i \sigma(R_i)\left[\Phi^{-1}(q) + \Phi^{-1}(0.5)\right] \end{aligned} \tag{3}$$

Let us first study the case in which all banks have the same exposure (β) to the common factor but differ in their idiosyncratic risk. We can then combine (2) and (3) as follows:

$$\begin{aligned} \Delta CoVaR_q^{S,i} &= d_i \sigma(R_i)[\Phi^{-1}(q) + \Phi^{-1}(0.5)] = d_i \sigma(R_i)\Phi^{-1}(q) \\ &= \frac{1}{N}\left(1 + \beta_i \sum_{j\neq i}\beta_j \frac{\sigma^2(F)}{\sigma^2(R_i)}\right)\sigma(R_i)\Phi^{-1}(q) \\ &= \left(\frac{1}{N}\sigma(R_i) + \frac{N-1}{N}\beta^2 \frac{\sigma^2(F)}{\sigma(R_i)}\right)\Phi^{-1}(q) \end{aligned} \tag{4}$$

If a bank increases its idiosyncratic risk, there are two opposing effects on ΔCoVaR: As captured in the first term within the parentheses, an increase in idiosyncratic risk increases ΔCoVaR. Since the bank is part of the system, the system co-moves with the idiosyncratic risk of bank i, which is reflected in ΔCoVaR. This effect becomes smaller, the lower the weight of the bank within the system is.

As captured in the second term within the parentheses, an increase in idiosyncratic risk decreases ΔCoVaR because higher idiosyncratic risk means that the bank's return contains less information about the system. This effect becomes larger, the lower the weight of the bank within the system is.

How ΔCoVaR is affected by an increase in idiosyncratic risk therefore depends on the composition of the system as well as on other parameters, such as the relative magnitude of factor risk and idiosyncratic risk. In Figure 1, we show the CoVaR for two exemplary banks that differ in their idiosyncratic risk. The choice of parameters is meant to be typical of daily returns. Qualitatively, results would not be affected if we scaled returns to other horizons. Specifically, we choose uniform beta values of 1 and the following volatility parameters (stated as per annum figures): $\sigma\left(F\right)=0.2$, $\sigma\left(\varepsilon_i\right)=0.2$ for $N-1$ banks, and $\sigma\left(\varepsilon_i\right)=0.4$ for one bank. To translate the parameters to daily returns, we divide by the square root of 260.

The figure shows that a higher idiosyncratic risk can have an ambiguous impact on ΔCoVaR. For a small number of banks, the bank with the larger idiosyncratic risk has a larger systemic risk according to ΔCoVaR because effect (i) described above dominates effect (ii). Once the number of banks is larger than three, the picture reverses. In this region, ΔCoVaR suggests that the bank with the higher idiosyncratic risk has a lower systemic risk.

What do these results imply for the usefulness of ΔCoVaR as a measure of systemic risk? It seems plausible that higher idiosyncratic risk should be captured by ΔCoVaR if the bank is so large that its idiosyncratic risk affects the system. For many banks, on the other hand, the second effect is likely to dominate in practice. It will be in the interests of those banks to increase their idiosyncratic risk because they will then be judged to have lower systemic risk. It is doubtful whether it is beneficial for system stability if the systemic risk measure used by regulators creates an incentive for banks to increase their idiosyncratic risk.

With respect to systematic risk, ΔCoVaR gets it right. Inspection of equation (4) shows that an increase in beta makes the bank have higher systemic risk as judged by ΔCoVaR.

We now turn to what Adrian and Brunnermeier (2011) call *exposure ΔCoVaR*. For this measure, we condition R_i on R_s rather than R_s on R_i. In our analysis, we therefore study

$$R_i = a_i + b_i R_s + u_i,$$

and obtain for a uniform β:

$$\begin{aligned} b_i &= \frac{\operatorname{cov}(R_S, R_i)}{\sigma^2(R_s)} = \frac{\beta^2\sigma^2(F) + N^{-1}\sigma^2(\varepsilon_i)}{\beta^2\sigma^2(F) + N^{-2}\sum_j \sigma^2(\varepsilon_j)} \\ a_i &= E(R_i) - b_i E(R_s) \\ \sigma^2(u_i) &= \sigma^2(R_i) - b_i^2\sigma^2(R_s) \end{aligned} \tag{5}$$

When we use the exposure ΔCoVaR to study the extent to which bank i is affected by the system, we obtain:

$$\begin{aligned}\Delta CoVaR_q^{i,S} &= \left[a_i + b_i\Phi^{-1}(q)\sigma(R_s) + \Phi^{-1}(q)\sigma(u_i)\right] \\ &\quad -\left[a_i + b_i\Phi^{-1}(0.5)\sigma(R_s) + \Phi^{-1}(q)\sigma(u_i)\right] \\ &= b_i\sigma(R_s)\left[\Phi^{-1}(q) + \Phi^{-1}(0.5)\right] = b_i\sigma(R_s)\Phi^{-1}(q)\end{aligned} \tag{6}$$

Let us again study the case in which all banks have the same exposure (β) to the common factor but differ in their idiosyncratic risk. Inserting (5) into (6) leads to:

$$\begin{aligned}\Delta CoVaR_q^{i,S} &= b_i\sigma(R_s)\Phi^{-1}(q) \\ &= \frac{\beta^2\sigma^2(F) + N^{-1}\sigma^2(\varepsilon_i)}{\beta^2\sigma^2(F) + N^{-2}\sum_j \sigma^2(\varepsilon_j)}\sigma(R_s)\Phi^{-1}(q)\end{aligned} \tag{7}$$

Standard calculus shows that b_i, i.e. the fraction in (7), increases if any $\sigma^2(\varepsilon_i)$ of the single idiosyncratic risks increases. The same holds for $\sigma(R_s)$. As $\Phi^{-1}(\alpha)$ is negative for the α of interest, the whole (negative) $\Delta CoVaR_\alpha^{i,S}$ is falling in $\sigma^2(\varepsilon_i)$. Hence, there is now only one effect on ΔCoVaR: An increase in idiosyncratic risk increases the systemic risk attributed by ΔCoVaR. The intuition is that higher idiosyncratic risk means that the system's return contains less information about the bank in question. This effect becomes weaker, the lower the weight of the bank within the system is. Thus, the problematic effect discussed above does not arise. As before, changes in systematic risk lead to the desired effect, i.e. the exposure ΔCoVaR attributes a higher systemic risk.

The next measure considered is the marginal expected shortfall (MES):

$$MES = E\left(R_i \middle| R_s < Q_S^\alpha\right)$$

As in exposure ΔCoVaR, a bank's return is conditioned on the system return. We therefore start by using the market model structure from above, $R_i = a_i + b_i R_s + u_i$, and obtain

$$\begin{aligned}MES &= E\left(R_i \middle| R_s < Q_S^\alpha\right) = E\left(a_i + b_i R_s + u_i \middle| R_s < Q_S^\alpha\right) \\ &= a_i + b_i E\left(R_s \middle| R_s < Q_S^\alpha\right) + E\left(u_i \middle| R_s < Q_S^\alpha\right)\end{aligned}.$$

By construction of an OLS regression, u_i and R_s are uncorrelated. Because their joint distribution is multivariate normal (both are linear images of independent normals), they are independent, so that $E\left(u_i \middle| R_s < Q_S^\alpha\right)$ is zero. We therefore obtain

$$MES = a_i + b_i\, E\left(R_s \middle| R_s < Q_S^\alpha\right)$$

where $E\left(R_s \middle| R_s < Q_S^\alpha\right)$ can be determined using familiar results for truncated normal distributions.

In Appendix A we show that adding idiosyncratic risk increases the systemic risk as measured by MES, as does an increase in systematic risk. The MES measure does not exhibit unwanted characteristics with respect to the choice of risk in the chosen setting.

The third measure that we focus on in this paper is the tail risk gamma. In the linear case studied in this section, the market model, which is nested in the tail risk gamma approach, provides the best possible description of a bank's return. The expected tail risk gamma is therefore zero irrespective of the choice of idiosyncratic risk and systematic beta risk, which means that we cannot derive statements on tail risk gamma properties within the setting of this section.

4 Systemic risk measures in the contagion case

After studying linear return relationships within a simple one-factor model, we now turn to examining contagion effects. An overview of different contagion definitions is given in Pericoli and Sbracia (2003). The main definition that we examine is one in which contagion is brought about by spillovers of idiosyncratic shocks.

Assume that the returns of the banks and the system evolve according to

$$\begin{aligned} R_1 &= \beta_1 F + \varepsilon_1, \\ R_j &= \beta_j F + \varepsilon_2 + \gamma I_{\{\varepsilon_1 < \kappa\}} \varepsilon_1, \quad j = 2, \ldots, N \\ R_s &= \frac{1}{N} \sum_j R_j \end{aligned} \tag{8}$$

That is, there is contagion from bank 1 to the other banks in the system. If bank 1 is afflicted by a realization of idiosyncratic risk that is worse than κ, other banks are partially affected, too. Other assumptions regarding the distribution of F and the ε_j are the same as above, i.e. they are independent normal variates. Since the dependence structure is now considerably more involved, we resort to Monte Carlo simulation to derive statements about systemic risk measures. To isolate the effects of contagion, we study a case in which the infectious and the infected banks do not differ in their variances. To this end, we need to determine the variance of $I_{\{\varepsilon_1<\kappa\}}\varepsilon_1$:

$$\begin{aligned}
\operatorname{var}\left(I_{\{\varepsilon_1<\kappa\}}\varepsilon_1\right) &= E\left(\left(I_{\{\varepsilon_1<\kappa\}}\varepsilon_1\right)^2\right)-\left(E\left(I_{\{\varepsilon_1<\kappa\}}\varepsilon_1\right)\right)^2 = E\left(I_{\{\varepsilon_1<\kappa\}}\varepsilon_1^2\right)-\left(E\left(I_{\{\varepsilon_1<\kappa\}}\varepsilon_1\right)\right)^2 \\
&= \Phi\left(\frac{\kappa}{\sigma(\varepsilon_1)}\right)\left[\operatorname{var}\left(\varepsilon_1 \middle| \varepsilon_1<\kappa\right)+\left(E\left(\varepsilon_1 \middle| \varepsilon_1<\kappa\right)\right)^2\right]-\left[\Phi\left(\frac{\kappa}{\sigma(\varepsilon_1)}\right)E\left(\varepsilon_1 \middle| \varepsilon_1<\kappa\right)\right]^2 \\
&= \Phi\left(\frac{\kappa}{\sigma(\varepsilon_1)}\right)\left[\operatorname{var}\left(\varepsilon_1 \middle| \varepsilon_1<\kappa\right)+\Phi\left(-\frac{\kappa}{\sigma(\varepsilon_1)}\right)\left(E\left(\varepsilon_1 \middle| \varepsilon_1<\kappa\right)\right)^2\right]
\end{aligned}$$

Both conditional moments can be calculated using familiar results for truncated normal variables.

We determine the systemic risk measures using 50 million simulated return observations for each bank, which also implies simulated values for the system return R_S. To increase precision, we use antithetic sampling for the factor returns and idiosyncratic shocks. When calculating ΔCoVaR, we need to condition on the VaR, which is observed with measure zero in the simulations. We therefore employ an interval around the VaR for conditioning. Specifically, we condition on the observations in the interval defined by the $(q-0.2\%)$ and $(q+0.2\%)$ quantiles of the simulated data. The $COVAR_{0.01}^{S|R_1=VaR_q^1}$, for example, is determined as follows: select the runs in which the return R_1 lies between the 0.8% and 1.2% quantile of R_1, and determine the 1% quantile of R_s for this selection. The exposure ΔCoVaR is determined accordingly.

The MES is determined as the average simulated return of a bank given that the simulated system return is below its 5% quantile.

The tail risk gamma is based on a regression of the simulated bank returns on the simulated system return and the change in out-of-the-money put options written on the system index. Similar to Knaup and Wagner (2012), we study options with a maturity of four months and a strike price equal to 70% of the index level. Put option prices are determined using Monte Carlo simulation and risk neutral valuation. We need to determine new put option values for the one-day-ahead index levels that occur in the simulations. With the parameters chosen here, there is no one-day system return outside the interval [–0.1, 0.1]. For each index value in [0.9×initial index value, (0.9+0.0001)×initial index, … , 1.1×initial value] we use Monte Carlo simulations to determine the value of a put option, whose maturity has by then changed to four months minus one day. For the analysis, the simulated index value is paired with the closest index value for which a put option price has been determined.

As in the previous section, the simulation is conducted using assumptions typical of daily returns. Beta values are uniform, $\beta_1 = \beta_j = 1$, and the following per-annum drift and volatility parameters are chosen: $\sigma(F) = 0.2$, $\sigma(\varepsilon_1) = 0.2$, $\sigma(\varepsilon_j) = \left(\sigma^2(\varepsilon_1) - \gamma^2 \operatorname{var}\left(I_{\{\varepsilon_1 < \kappa\}}\varepsilon_1\right)\right)^{0.5}$ for all $j > 1$, and $E(F) = 0.05$. The risk-free rate is set to 0.02. To translate the parameters to daily returns, we divide by 260 in the case of $E(F)$ and by the square root of 260 in the case of standard deviations. The number of banks is set to $N = 50$. The contagion threshold κ is set to either –0.0204, –0.0289 or –0.0383, corresponding to the 5%, 1%, and 0.1% quantile of ε_1. The responsiveness to contagion, γ, is set to either 0.75 or 0.25.

Results are reported in Table 1, which shows that ΔCoVaR measures do not provide a clear identification of contagious banks. Depending on the contagion threshold, the contagion intensity, as well as on the direction of the ΔCoVaR measure, the bank that is infectious can be assigned a lower or a higher ΔCoVaR than the bank that becomes infected. The difference is most pronounced in Panel A, which assumes strong contagion effects for idiosyncratic shocks below their 5% quantile, i.e. a case in which the region used for conditioning will include many instances of contagion.

This result is surprising since a strong idiosyncratic loss incurred by the infectious bank causes substantial losses for all other banks and hence for the system. Such a contagion can be expected to appear more often in the event $\mathfrak{R}_1 = \left\{R_1 = Q_\alpha\left(R_1\right)\right\}$ than in $\mathfrak{R}_j = \left\{R_j = Q_\alpha\left(R_j\right)\right\}$, where j denotes any of the infected banks. These are the conditioning events of the CoVaR. We would therefore expect the system's risk to be rather large when conditioning on $\mathfrak{R}_1$, compared to conditioning on $\mathfrak{R}_j$, where contagion is less frequent.

To get an intuition why we find the opposite, Panel A of Figure 2 plots the system return against the return of the infectious bank and the return of an infected bank, respectively, choosing parameters as in Panel A of Table 1. Conditional on the bank returns being at their 1% quantiles, the system return has a larger variance in the case of the infected bank, which leads to a more extreme ΔCoVaR. The explanation for the difference in the variances is found by splitting up the events $\mathfrak{R}_1$ and $\mathfrak{R}_j$ into the cases with and without contagion.

Panel B plots instances of contagion, which we subsume under $\mathfrak{C} = \{\varepsilon_1 < \kappa\}$. The left-hand graph shows a strong ($\mathfrak{C}$-conditional) correlation between R_1 and the system return so that, once R_1 is fixed at its quantile, R_S exhibits only low variation. The reason for the high correlation is that the bank's idiosyncratic risk has spread through the system. To show this formally, consider $\mathfrak{R}_1 \cap \mathfrak{C}$. The first line of (8) then gives $F = \beta^{-1}\left(Q_\alpha\left(R_1\right) - \varepsilon_1\right)$, which we can plug into the second line to eliminate F in the representation of R_j. Averaging over individual returns, the system return now reads

$$R_s = \beta Q_\alpha\left(R_1\right) + \left[\gamma \frac{N-1}{N} + \frac{1}{N} - \beta\right]\varepsilon_1 + \frac{1}{N}\sum_{j=2}^{N}\varepsilon_j \quad (9)$$

Its variance is low because the first term on the right-hand side is deterministic, the coefficient in brackets is small for large N given γ and β are not too different, while ε_1 has low variance under $\mathfrak{C}$ anyway, and the third term is diversified over independent risks and hence of low variance, too. In contrast, no such strong relationship between system and individual return exists if an infected bank is in distress. Rewriting the system return in the same manner as above gives, under $\mathfrak{R}_j \cap \mathfrak{C}$,

$$R_s = \beta Q_\alpha\left(R_j\right) + \left[\gamma\left(\frac{N-1}{N} - \beta\right) + \frac{1}{N}\right]\varepsilon_1 + \frac{1}{N}\sum_{i=2}^{N}\varepsilon_i - \beta\varepsilon_j \quad (10)$$

with a similar structure, except for the last addend. This term bears substantial variation relative to the others and is the reason why the standard deviation of R_s under $\mathfrak{R}_j \cap \mathfrak{C}$ is considerably larger than under $\mathfrak{R}_1 \cap \mathfrak{C}$.

Realizations without contagion are plotted in Panel C of Figure 2. While the graphs indicate differences in the joint conditional distributions, the latter are similar in the part relevant for the ΔCoVaR. To see this more clearly, consider the counterparts to (9) and (10) in the absence of contagion, which are

$$R_s = \beta Q_\alpha\left(R_1\right) - \left[\left(\beta - \frac{1}{N}\right)\varepsilon_1\right] + \frac{1}{N}\sum_{j=2}^{N}\varepsilon_j$$

for $\mathfrak{R}_1 \setminus \mathfrak{C}$ and

$$R_s = \beta Q_\alpha\left(R_j\right) - \left[\beta\varepsilon_j - \frac{1}{N}\varepsilon_1\right] + \frac{1}{N}\sum_{i=2}^{N}\varepsilon_i$$

for $\mathfrak{R}_j \setminus \mathfrak{C}$. The expressions in brackets in both formulas are not identical as ε_1 is bounded from below (we are in the non-contagion case) while ε_j is not. However, the distributions of ε_1 and ε_j are quite similar for positive realizations so that the conditional distributions of R_s are similar on the negative half, which is the part relevant for ΔCoVaR.

Having analyzed the risk of R_s with and without contagion separately, we now put both cases together. We observe that the low variance of R_s under $\mathfrak{R}_1 \cap \mathfrak{C}$ (compared to $\mathfrak{R}_j \cap \mathfrak{C}$) is not offset by an opposing relationship in the non-contagion case ($\mathfrak{R}_1 \setminus \mathfrak{C}$ vs. $\mathfrak{R}_j \setminus \mathfrak{C}$), where the relevant parts of the distribution are fairly similar. Even if contagion were equally frequent under $\mathfrak{R}_1$ and $\mathfrak{R}_j$, we would thus observe that the CoVaR of an infected bank is more negative than the CoVaR of the infectious bank. In fact, contagion is more frequent under $\mathfrak{R}_1$ so that the total effect is even stronger.

As demonstrated by this example, contagion can cause complex return patterns which make it difficult to identify the infectious bank with a CoVaR-type measure.

One might suspect that the ambiguities associated with CoVaR arise from problems associated with value at risk, and that they can be eliminated by moving to co-expected shortfall (CoES). However, further analysis shows that this is not the case. For example, when we implement the ΔCoES as suggested by Adrian and Brunnermeier (2011) for the parameters of Panel A, the ΔCoES is –1.92% for the infectious bank and –2.57% for the infected bank.

In each of the cases studied here, MES and tail risk gamma, by contrast, assign a higher systemic risk to the infectious bank. Both measures appear to be more robust because they do not focus on a single quantile of a distribution but on a range of quantiles. If contagion leads to extremely negative returns, MES will pick it up because it averages across the bad days of the market, while the tail risk gamma picks up the concavity that is generated by the contagion effects.

There might be other contagion structures in which these measures provide different rankings of contagious and infected banks. We have examined the following structures and found similar effects as in Table 1:

(i) We modeled volatility spillovers: If the idiosyncratic risk of the contagious bank falls below κ, the idiosyncratic risk of the infected banks is *m* times higher than in the base case. The base case volatility is again chosen such that the total volatility of the infected banks equals the one of the infectious bank. Assuming $\kappa = -2.04\%$ and $m = 3$, which we choose based on return behavior surrounding the Lehman crash,[4] the ΔCoVaR values are –2.09% and –2.36% for the infectious and an infected bank, respectively. MES assigns a higher risk to the infectious bank.

(ii) We modeled a spillover of returns rather than a spillover of idiosyncratic risk: If the return of bank 1 falls below κ, an amount of $\gamma\kappa$ is added to the return of the infected banks. With $\kappa = -2.89\%$ and $\gamma = 0.75$ the simulated ΔCoVaR values are –5.07% and –5.38% for the infectious and an infected bank, respectively. MES now also assigns a higher risk to the infected banks.

It is worth noting that the differences in MES and tail risk gamma appear to be relatively small in some cases. Given that the infectious bank in the example has a strong influence on other banks, one would perhaps expect larger discrepancies. The MES difference of –3.10% versus –2.94% in Panel A of Table 1 has the same size as the MES difference between two non-infectious banks that have betas of 1 (MES = –2.74%) and 1.07 (MES = –2.56%), respectively. While regulators would probably view the infectious bank with great suspicion (if they knew about the infection mechanism as modeled here), they probably would not pay the same attention to a beta difference of 0.07.

We conclude by noting that it is not necessarily obvious from a regulatory perspective whether a systemic risk measure should assign a higher risk to contagious banks. In the presence of contagion, financial stability can be increased by imposing stricter standards on contagious banks, thus reducing the likelihood and magnitude of contagious events. However, it could also be increased through stricter standards for infection-prone banks, i.e. by reducing the consequences of a contagious event. The lesson to be drawn from this section is therefore not so much that some measures correctly identify contagion while others do not. Rather, it shows that risk measures can provide conflicting risk rankings, and that even simple contagion structures can be difficult to identify.[5]

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[4] For the 29 depositary institutions listed in Acharya et al. (2010, Appendix A), we examined the idiosyncratic volatility over the 30 days ending on September 12, 2008 as well as over the 30 days starting on September 15, 2008 (Lehman collapse) . Using a one-factor model with the S&P 500 as the factor, the median idiosyncratic volatility increases by a factor of 2.98.

[5] Identification problems also arise in the standard correlation-based analysis of contagion, cf. Pesaran and Pick (2007).

5 Empirical robustness of tail-risk measurement

5.1 Measurement issues

The main interest of financial regulators lies in extreme events. Acharya et al. (2010, p.15) "think of systemic events […] as extreme tail events that happen once or twice a decade (or less), say." On a daily frequency, this corresponds to events occurring with a probability of less than 0.1%. In a typical sample available for estimation, such events are either not observed, or their number is so small that it is difficult to base statistical inference on the extreme events only. The literature is aware of these problems and suggests the following solutions.

Acharya et al. (2010) examine events that occur with a probability of 5%. With the help of extreme value theory and assuming power law distributions, they show that the ultimate object of interest, which they call systemic expected shortfall, is linearly related to the 5% MES as well as other variables. If return distributions are similar and of the kind assumed by Acharya et al., this promises that the order of MES_i in a group of banks does not change strongly if the tail probability is raised from, say, 0.1% to 5%. Adrian and Brunnermeier (2011) suggest using a large sample of historical returns in order to include as many extreme events as possible. Since the risk characteristics of financial institutions can change over time, they study not only unconditional risk estimates, but also estimates of the conditional distribution as a function of state variables. Knaup and Wagner (2012), finally, try to circumvent the data problem by utilizing the information contained in put prices.

For the following reasons, some skepticism appears to be in order. The theoretical results presented by Acharya et al. (2010) rest on the assumptions made in their paper and do not hold in general. We will show that the relationship between MES and extreme risk can break down if portfolios include standard option positions. Adrian and Brunnermeier (2011) face a similar challenge. Even 25 years of data may not be enough to identify co-movement in very extreme scenarios. Note that in the examples studied here, there is no time variation in risk parameters, which is why we do not model the conditional distribution. The use of put options in Knaup and Wagner (2012), finally, expands the information set by including market expectations about extreme events. If such expectations do not change significantly over the estimation sample, however, there is again only little information that can be used to estimate the object of interest. As the examples show, the tail risk gamma of Knaup and Wagner (2012) faces a problem similar to that of the other two measures. Gamma estimates may predominantly be based on less extreme changes in market expectations, and there is no guarantee that there is a robust link between a portfolio's sensitivity to very extreme and less extreme changes, respectively.

5.2 Some archetypical portfolio structures

We examine the risk measures' robustness by determining them for a number of portfolios that differ in tail risk. These differences are engineered through positions in standard options as well as differences in beta risk. For simplicity, we perform calculations in a Black-Scholes world with lognormally distributed market index returns. We assume a risk-free rate of 2% and a stock market volatility of 20%. To approximate individual portfolios, we assume an idiosyncratic volatility of 20%. Portfolios differ in their betas and in option positions. Without options, the portfolio return of institution i from $t-1$ to t would be

$$R_{it} = R_f + \beta_i \left(R_{mt} - R_f \right) + \varepsilon_{it} \,,$$

where R_f denotes the risk-free rate, R_m the market return, and ε_{it} is the idiosyncratic risk with a variance of $0.2^2 / 260$. We refer to a portfolio without options and $\beta = 1$ as the *baseline portfolio*.

At the end of each day, various put options with a maturity of 30 days are bought or sold, indexed by j and each with a (positive or negative) weight w_{ij} relative to the portfolio value without options. If aggregate option weights are positive, funding is obtained at the risk-free rate; otherwise the obtained cash is invested in the risk-free asset. The option positions are unwound at the end of the following day.

Let $p\left(S, K \times S, d\right)$ be the Black-Scholes price of a put option if the price of the underlying is S, for a time to maturity of d days and a strike price $K \times S$. Normalizing the index price to one, the return of a portfolio from $t-1$ to t obtains as

$$R_{it} = \left(1 - \sum_j w_{ij} \right) R_f + \beta_i \left(R_{mt} - R_f \right) + \sum_j w_{ij} \left(\frac{p\left(1 + R_{mt}, K_{ij}, 29\right)}{p\left(1, K_{ij}, 30\right)} - 1 \right) + \varepsilon_{it}$$

We analyze the performance of systemic risk measures for four archetypical pay-off structures, denoted by A to D. In each setting, there are 16 portfolios which share the same archetypical structure but differ in their risk. The first portfolio is always equal to the baseline portfolio.

In setting A, portfolios do not contain options. They differ in their betas, which linearly increase from $\beta = 1$ for portfolio A1 to $\beta = 2$ for A16. Figure 3 plots the market return against expected portfolio returns, conditional on the market return, for portfolio A16 and the portfolios No. 16 of the other types.

In setting B, portfolio No. 16 has short positions in out-of-the-money put options (weight -0.45%, strike 0.8), which are balanced with long positions in at-the-money put options (weight 3%). This produces a concave return profile (Figure 3). Compared to the market portfolio (gray line), the portfolio generates smaller losses on days with a moderately negative market return, leaving aside idiosyncratic risk. Assuming a drift rate of 5%, the 5% quantile of the market return is –2%. In such an event, the portfolio performs better than the market. Its performance drops below the market when the market return hits –2.5%. With the assumed distribution, this happens with a probability of 2.1%. On the way from portfolio B16 to portfolio B1, option weights are linearly reduced to zero while betas linearly step down to 1.

In setting C, out-of-the-money put options (weight 0.75%, strike 0.8) provide protection or even overprotection against large losses, while an increase in betas provides upside participation. Due to its high beta and the put being far out of the money, Portfolio C16 is comparable to the baseline portfolio with a leveraged systematic component, as long as returns are moderate. Observing such moderate returns suggests that the portfolio is riskier than in the baseline. If losses in the market index are large, however, the protection provided by the put overcompensates losses in the market component (at a probability of 1.23%, if $R_m < -2.8\%$) and can even generate profits (with a probability of 0.06%, if $R_m < -4\%$). On the way from portfolio C16 to portfolio C1, option weights are linearly reduced to zero, while betas are linearly reduced to 1.

In setting D, two put options are included, and the beta of 1.375 for portfolio D16 is only slightly increased against the baseline portfolio. The long put position that is less far out of the money (0.725) has quite a heavy weight of 5.7% and so sets a nearly perfect floor on the losses in the systematic component. This works as long as R_m does not fall short of -2.8%.

If it does, the second, short put option position (strike 0.7, weight –4.5%) gains impact and generates dramatic losses. A key difference between the profiles of setting D and those of the other settings is that the return profiles in D are not globally concave or convex. This complicates inference about extreme losses. On the way from portfolio D16 to portfolio D1, option weights are again linearly reduced to zero, while betas are linearly reduced to 1.

Errors in estimating the systemic tail risk of portfolios can arise for various reasons. First, the relationship between portfolio payoffs and systemic factors may be so nonlinear and even non-monotonic that risk measures for different loss severities are only loosely connected. Second, estimations can be noisy due to sampling error, and, third, the estimators may be misspecified, for instance, by assuming nonlinear dependencies to be linear. We examine these errors separately. In Section 5.3, we use *precise* MES and CoVaR under varying tail probabilities to analyze how well low-probability risk measures can be inferred from medium-probability ones. As estimation errors are excluded, the results provide an upper bound for the quality that can be achieved in reality. In Section 5.4, we perform a simulation exercise where MES, CoVaR, and tail risk gamma are estimated under realistic conditions.

5.3 Comparing exact risk measures for different confidence levels

For all portfolios from A1…A16 to D1…D16 we calculate precise MES and both types of ΔCoVaR at confidence levels of 0.1%, 1%, 5%, and 10%. With our assumptions, the exposure ΔCoVaR can be computed analytically, while MES and ΔCoVaR are determined through numerical integration. The 0.1% confidence represents the object of interest as it corresponds to a probability with which systemic events occur. A 1% confidence is used in the ΔCoVaR estimations of Adrian and Brunnermeier (2011), while Acharya et al (2010) suggest choosing the 5% MES as a proxy for the unobservable systemic expected shortfall. The tail risk gamma approach of Knaup and Wagner (2012) does not lend itself easily to a similar exercise. We could study gammas for different choices of moneyness. However, Knaup and Wagner already choose options that are deep out-of-the-money, corresponding to very extreme events.

Separately for each setting, and singling out four portfolios, exact MES and ΔCoVaR for different confidence levels are presented in Figure 4. Not surprisingly, risk measures of the portfolios in setting A are in the expected order according to the varied beta. In Appendix C we demonstrate that an extrapolation from 1%, 5%, and 10% to 0.1% performs well.

Things are different for setting B. Running from B1 to B16, the MES curve more or less rotates around the values for a confidence level of 1%. The variation in the 0.1% MES is strongest. This "rotation" means that the proposal by Acharya et al. (2010) to choose the 5% MES as a proxy would fail; precisely the wrong order would be predicted. The result for the exposure ΔCoVaR is very similar. The shape of the curves suggests trying an extrapolation: assuming that the MES at 1% or higher could be estimated with great precision, it seems that an extrapolation might work well because the convexity of each curve at 5% nicely corresponds with the convexity at 1% (see Appendix C).

The ΔCoVaR does not fit into the pattern. It assigns the lowest systemic risk to the most risky portfolio B16 – but now on all confidence levels. To get an intuition why, let us mimic the concave structure of the portfolio by a simpler, piecewise linear profile with a kink in k:

$$R_i^{\text{kink}} = \begin{cases} \beta_i R_m + k\left(1-\beta_i\right) + \varepsilon_i & : \quad R_m < k \\ R_m + \varepsilon_i & : \quad R_m \geq k \end{cases} \tag{11}$$

We assume in this example, for ease of exposition, that returns are not lognormally but normally distributed, with zero drift and equal variance: $R_m : N\left(0,\sigma^2\right)$ and $\varepsilon_i \sim N\left(0,\sigma^2\right)$. This is a very small adjustment.[6]

If β_i is larger than 1, the profile is similar to a buying portfolio with a beta of 1, plus writing short-term put options with strike k.

We compare the ΔCoVaR of this kinked profile to a portfolio that is linear in the market portfolio, with a beta of one and the same idiosyncratic risk as in the kinked portfolio. The return on this linear portfolio is thus $R_m + \varepsilon_i$ and is denoted R_i^{lin}. As in Section 3, we derive an orthogonal representation of the market risk factor $R_m = 0.5 R_i^{\text{lin}} + \nu$ with $\sigma\left(\nu\right) = \sqrt{2}/2\,\sigma$ (here we utilize $\sigma \equiv \sigma\left(R_m\right) = \sigma\left(\varepsilon_i\right)$ and hence $\sigma\left(R_i^{\text{lin}}\right) = \sqrt{2}\sigma$). Conditioning on R_i^{lin} is now straightforward and provides us with

$$Q_\alpha\left(R_m \mid R_i^{\text{lin}} = Q_\alpha\left(R_i^{\text{lin}}\right)\right) = \frac{1}{2} Q_\alpha\left(R_i^{\text{lin}}\right) + Q_\alpha\left(\nu\right) = \frac{\sqrt{2}}{2}\sigma\Phi^{-1}\left(\alpha\right) + Q_\alpha\left(\nu\right)$$

When we condition on the median, this simplifies to:

$$Q_\alpha\left(R_m \middle| R_i^{\text{lin}} = Q_{0.5}\left(R_i^{\text{lin}}\right)\right) = \frac{1}{2} Q_{0.5}\left(R_i^{\text{lin}}\right) + Q_\alpha\left(\nu\right) = Q_\alpha\left(\nu\right) \tag{12}$$

where $Q_{0.5}\left(R_i^{\text{lin}}\right) = 0$ so that

$$\Delta CoVaR\left(R_m \middle| R_i^{\text{lin}}\right) = \frac{\sqrt{2}}{2}\sigma\Phi^{-1}\left(\alpha\right) \approx 0.71 Q_\alpha\left(R_m\right). \tag{13}$$

[6] Daily returns with 20% (annual) volatility have such a small standard deviation that the lognormal and normal distributions are very similar. The largest deviation between distribution characteristics relevant here occurs between the 0.1% quantiles; they differ by factor 1.02.

Now consider an extreme case in which the β_i of the kinked portfolio is very large and k is in the left tail of the distribution of R_m but less extreme than the α-quantile. In this "kink" case it holds approximately that

$$Q_\alpha\left(R_m \left| R_i^{\text{kink}} = Q_\alpha\left(R_i^{\text{kink}}\right)\right.\right) \approx Q_\alpha\left(R_m\right)$$

because the distribution of R_i^{kink} in the left tail is almost fully determined by the market return if β_i is large. If R_i^{kink} is at its α-quantile, we can then infer almost perfectly that the market is at its α-quantile as well.

For determining the CoVaR conditional on R_i^{kink} being at its median, we distinguish two cases:

In *case one*, the condition that the portfolio return must take on its median value compresses the distribution of the market return more or less completely onto the range above the kink. This is true if

$$\Pr\left(R_m < k \left| R_i^{\text{lin}} = Q_{0.5}\left(R_i^{\text{lin}}\right)\right.\right) = \Pr\left(\nu < k\right) = \Phi\left(\frac{\sqrt{2}k}{\sigma}\right) = \alpha \tag{14}$$

Then it does not matter much for the CoVaR whether there is some kink below the relevant range so that R_i^{kink} and R_i^{lin} have a very similar CoVaR of approximately $Q_\alpha\left(\nu\right)$. Taking (12) into account, we conclude

$$\Delta CoVaR\left(R_m \left| R_i^{\text{kink}}\right.\right) \approx Q_\alpha\left(R_m\right) - Q_\alpha\left(\nu\right) \approx 0.29\, Q_\alpha\left(R_m\right),$$

which is less extreme than the ΔCoVaR in the linear benchmark case, despite the fact that the downward kink can imply considerable exposure to systematic tail risks.[7]

In *case two*, (14) is not fulfilled so that the kink does play as role for the CoVaR calculations. Then we find the approximation

$$Q_\alpha\left(R_m \left| R_i^{\text{kink}} = Q_{0.5}\left(R_i^{\text{kink}}\right)\right.\right) \approx k$$

[7] For parameters that bring the kinked profile close to portfolio B16 (for instance, $k = -0.035$, $\beta_i = 7$), .we determined the 0.1% ΔCoVaR through simulations; they confirmed that the approximation correctly captures the ordering of the ΔCoVaR.

which holds because R_i is very unlikely to be at its median if R_m is below k. If R_m were below k, the high beta would push R_i so far away from its median that idiosyncratic risk could not bring it back, except in very rare cases. ΔCoVaR thus approximately is

$$\Delta CoVaR\left(R_m \left| R_i^{\text{kink}}\right.\right) \approx Q_\alpha\left(R_m\right) - k \ ,$$

the size of which can now exceed the one in the linear case: if α shrinks, ΔCoVaR in the kink case decreases at the same speed as $Q_\alpha\left(R_m\right)$, in contrast to $0.71 Q_\alpha\left(R_m\right)$ in the linear case, as shown in (13). But still there is a substantial range, approximately if $k < \left(1-0.71\right) Q_\alpha\left(R_m\right)$, where the ΔCoVaR of R_i^{kink} has a smaller absolute value.

To summarize our example, adding a downward kink to the portfolio's market sensitivity in the lower tail may "confuse" the ΔCoVaR in an undesirable way: less systemic risk is the consequence of more downward exposure. In Appendix B, we generalize the kink example to smooth nonlinearities. When the risk profile is established by a monotonic function h so that $R_i = h\left(R_m\right) + \varepsilon_i$, we argue that the CoVaR is almost exclusively driven by the steepness of the profile in the lower tail, i.e., by the derivative $h'\left(Q_\alpha\left(R_m\right)\right)$, and often in the undesirable way, as observed for the kink example.

The sequence of portfolios in setting C gives a similar picture as in setting B: risk measures switch the order if the tail probability is changed, but the MES switches between points other than those where CoVaR switches. At the 1% level, MES and CoVaR give a correct prediction for the order of 0.1% risk measures. If 1% estimates are not accessible, so that one has to rely on higher probability levels, capital surcharges based on such measures would create wrong incentives – the larger the hedge in the tail, the more it would be "punished" by increased capital requirements. Similar to setting B, extrapolation works well (see Appendix C).

If the portfolio profile is more complex, as is the case with setting D, prospects for inferring the 0.1% risk measures from the others are even worse. The order of risk measures switches below a 1% tail probability and hence within a de facto unobservable region. While the MES curves still seem to be accessible to extrapolation (note that variation in convexity at 5% is low, however), this cannot be said about the CoVaR curves. They are virtually straight between 1% and 10%, so that little if nothing can be learned from these points about the convexity at 1%. In Appendix C we show that our extrapolation fails.

It is remarkable that, as in setting B, ΔCoVaR assigns the lowest risk to the portfolio with the largest short-put positions. Due to the trough-shaped profile for moderately negative returns, the effects are more involved than before, and the CoVaR under the median condition contributes as much to the effect as the one under the tail condition (unreported). However, the arguments of Appendix B can be applied to the latter at least: when conditioning on an extreme quantile, the CoVaR is almost exclusively driven by the steepness of the profile in the lower tail, and the CoVaR's size may shrink while the profile in the tail becomes steeper. This happens in the transition from portfolio D1 to D16.

5.4 Estimation under realistic conditions

To examine how well the four measures would discriminate between low-risk and high-risk portfolios in a practical application, we conduct a simulation study. To facilitate interpretation, the analysis is conducted separately for each portfolio setting (A, B, C, or D). The analysis has the following structure:

(1) Choose one of the four portfolio settings A, B, C, or D.

(2) For the market return and the 16 portfolios of the setting chosen in (1), simulate daily returns using the assumptions and portfolios from Section 5.2. For the ΔCoVaR computations, aggregate five daily returns to one weekly return.

(3) Determine the risk measures in line with the choices made in the literature:

MES:	260 days, 5% confidence level
ΔCoVaR:	1,300 weeks, 1% confidence level
Exposure ΔCoVaR:	1,300 weeks, 1% confidence level
Tail risk gamma:	260 days, put with maturity 4 months and strike 70%

(4) Repeat steps (1) to (3) 1,000 times.

Figure 5 plots average estimated risk ranks across the 1,000 trials, along with 90% confidence intervals, against the true risk rank. A perfect system, which always identifies the correct systemic risk, would imply a diagonal performance line with a zero-width confidence interval. For each of the four measures, the true risk rank is based on the 0.1% MES or 0.1% *exposure* ΔCoVaR, which generate the same ranking. The previous section has shown that ΔCoVaR reverses these rankings for portfolio settings B and D, but it has also shown that this reversal should be attributed to a particularity of the ΔCoVaR definition rather than to differences in systemic risk. For this reason, we compare the estimated ΔCoVaR ranks against the true risk ranks implied by MES and exposure ΔCoVaR. We do the same for the tail risk gamma, which is an empirical measure that involves the choice of certain put options; it cannot be used directly to infer a correct risk ranking for some confidence level.

In Setting A, in which portfolios differ only by their beta, the MES and the two ΔCoVaR measures manage to identify the correct risk rank on average. The width of the 90% confidence intervals is around five for MES and exposure ΔCoVaR, meaning that most incorrect ranks differ by less than +/– 3. ΔCoVaR provides a less reliable ranking but also gets the ranking right on average. In contrast, the performance line of the tail risk gamma is flat, which implies that a random ordering would perform equally well. Since the gamma measures tail risk after controlling for market beta, however, this is what we would expect. The return profile is linear, so there is nothing left that could be detected by the gamma. When combined with the standard beta estimate, the gamma would result in a correct ordering.

In summary, the results from the first setting appear promising; and the confidence intervals give a good benchmark for the magnitude of the estimation error that we would expect under ideal conditions, i.e. if return profiles are linear in the market.

The picture changes when portfolios with options are examined. Recall that in setting B, a short position in out-of-the-money puts is combined with a long position in at-the-money puts. MES as well as the two CoVaR measures typically order the portfolios in the wrong way. On average, the least risky portfolio is judged to be the portfolio with the highest risk. The extent of misestimation is most pronounced for exposure ΔCoVaR. The explanation is straightforward. Portfolios with larger option weights have a larger tail risk, but when moderately negative, their returns are less pronounced than the market returns. The confidence levels defining the MES and ΔCoVaR chosen for the estimation are not extreme enough to capture what is going on in the tail. The weak discriminatory power is consistent with Figure 4 where the exact 5% MES of all B portfolios are close neighbors (similar for exposure CoVaR). The tail risk gamma, on the other hand, does a good job in discriminating risks. The return profile is concave over the entire domain; as is the profile of the put option that is used in the regression. The tail risk gamma can therefore provide a good estimate of the portfolio’s curvature, which in turn is monotonically related to the tail risk. When taking the width of the confidence intervals into account, however, the performance of the rail risk gamma is inferior to the one of the well-performing measures in the linear case.

In setting C, portfolios differ in their protection. The portfolios have a J-shaped return profile. The relative performance is similar to setting B. Exposure ΔCoVaR performs worst, followed by MES and ΔCoVaR. Again, this conforms to Figure 4 where all 5% MES for C portfolios almost coincide. The explanation for this pattern is analogous to setting B. In setting C, return profiles are convex rather than concave. Due to the high confidence levels, MES and ΔCoVaR overestimate the risk of portfolios with a high curvature. The tail risk gamma performs well, and better than in setting B. This can be explained by noting that the portfolios in B contain two options, while the C portfolios contain only one option. As the regression used for the estimation of the tail risk gamma contains only one option, the curvature estimates derived from the regression are better in the C setting.

The final panel squashes the hopes that there always is at least one measure that provides a good ranking of risks. The return profiles of the D portfolios are concave in the extreme tail but convex in the remaining part. The estimation methods mainly use observations from the convex part, and therefore do not get the ordering right. As is evident from Figure 4 and the return profiles shown in Section 5.2, errors would be economically significant. The high-risk portfolios, which are mostly judged to be the least risky by the four measures, embody much greater tail risk.

6 Conclusion

We started our analysis of return-based systemic risk measures by examining whether they adequately indicate differences in systematic risk, idiosyncratic risk, or contagion. The *ΔCoVaR* measure suggested by Adrian and Brunnermeier (2011) assigns a lower systemic risk as idiosyncratic risk is increased. In the examples studied here, it also assigns a higher systemic risk to infected banks, whereas marginal expected shortfall and tail risk gamma mostly do the opposite.

We then explored how limited data availability typical of practical applications may inhibit the measures' performance. Typical data sets available for estimating systemic risk do not include a sufficient number of extreme events. Hence, relying on observations of less extreme events is unavoidable. We use positions in standard index options to illustrate that this can lead to serious misestimation of systemic risk. It is possible to take large tail risks that remain nearly invisible in the estimated risk measures. To make matters worse, options can be used to diminish losses in regions from where most of the data for the empirical risk measures are collected, so that systemic risk appears to be very low even though it is extremely high. On the other hand, protective put strategies that are immune to extreme shocks can be judged to have high systemic risk because estimation methods rely on less extreme return realizations in which option premia depress returns relative to unprotected institutions.

Taking liquidity and transaction costs into account, the option structures examined in the paper may not be feasible for large financial institutions. However, non-linearities can also arise from loan exposures, credit derivatives or bespoke equity derivatives. The key insight from the analysis is that non-linearities can have a large impact on the informativeness of systemic risk measures.

Some improvement might be achieved by modeling the relationship between extreme and less extreme quantiles. For example, it would be possible to calculate a range of risk measures based on tail probabilities between 1% and 10%, laying a smooth curve through them, and evaluating them at 0.1%. However, whatever the extrapolation method is – once market participants know it, they might be able to dupe it by bespoke derivatives positions.

Together, these observations raise doubts about the informativeness of the proposed measures. In particular, a direct application to regulatory capital surcharges for systemic risk could create wrong incentives for banks. We conclude that regulatory capital surcharges for systemic risk should not rely exclusively on market-based measures of systemic risk, and that more work needs to be done in order to assess the reliability of information that can be drawn from a return-based analysis of systemic risk.

Appendix

A Sensitivity analyses of the MES

We are interested in the sensitivity of MES under the linear model of Section 3 to the idiosyncratic risk of a single bank, leaving everything else constant. Idiosyncratic risk is measured by $\sigma^2(\varepsilon_i)$. The MES in the linear model is given by

$$MES = a_i + b_i E\left(R_s \middle| R_s < Q_S^\alpha\right)$$

We notice $E(R_s) = \beta\mu$ and rewrite the conditional expectation to

$$E\left(R_s \middle| R_s < Q_S^\alpha\right) = \beta\mu + \sigma(R_s) E\left(Z \middle| Z < Q_Z^\alpha\right)$$

where Z is standard normal. The expectation on the right-hand side depends on α only and is negative for the α of interest. Setting $C^{\alpha} \equiv E\left(Z \middle| Z < Q_Z^{\alpha}\right)$ we obtain

$$MES = a_i + b_i\left(\beta\mu + \sigma\left(R_s\right)C^{\alpha}\right) \tag{A.1}$$

Standard calculus shows that the partial derivative of b_i to $\sigma^2\left(\varepsilon_i\right)$ is positive. Because $\sigma\left(R_s\right)$ grows with $\sigma^2\left(\varepsilon_i\right)$ and C^{α} is negative, the term in parentheses decreases; it is also negative for relevant parameter choices, as can be seen from the estimate

$$\beta\mu + \sigma\left(R_s\right)C^{\alpha} = \beta\mu + \sqrt{\beta^2\sigma^2\left(F\right) + N^{-1}\sum\sigma^2\left(\varepsilon_i\right)}C^{\alpha} \le \beta\left(\mu + \sigma\left(F\right)C^{\alpha}\right),$$

again taking into account that C^{α} is negative. As $C^{0.05}$ is already smaller than –2 and even more negative for more extreme confidence levels, we would have to have a market where the index has a drift more than twice as large as its volatility. This is very unusual. Assuming that the parentheses term in (A.1) is indeed negative, the growing b_1 makes the magnitude of the MES grow when idiosyncratic risk rises.

B Sensitivity analysis of ΔCoVaR

To gain more insight into the way ΔCoVaR may depend on option positions, we use the market model of Section 5.2 and replace the example of a kinked risk profile as in (11) by a smooth profile. We assume that the sensitivity of the portfolio return on the market return is expressed by

$$R_i = h\left(R_m\right) + \varepsilon_i\,, \tag{B.1}$$

where h is a smooth, strictly increasing function, as it is given for the portfolio types A and B.

First, we look at the case of zero idiosyncratic risk. This turns R_m and R_i into co-monotonous variables so that their quantiles are strictly linked: $Q_{\alpha}\left(R_i\right) = h\left(Q_{\alpha}\left(R_m\right)\right)$. Under $\left\{R_i = Q_{\alpha}\left(R_i\right)\right\}$, the market return is deterministic, and we have, introducing $h^{(-1)}$ as the (also strictly increasing) inverse of h,

$$\begin{aligned}\Delta CoVaR_\alpha^i &= h^{(-1)}\left(Q_\alpha\left(R_i\right)\right) - h^{(-1)}\left(Q_{1/2}\left(R_i\right)\right)\\ &= h^{(-1)}\left(h\left(Q_\alpha\left(R_m\right)\right)\right) - h^{(-1)}\left(h\left(Q_{1/2}\left(R_m\right)\right)\right)\\ &= Q_\alpha\left(R_m\right) - Q_{1/2}\left(R_m\right).\end{aligned} \tag{B.2}$$

Since h has no effect on the distribution of R_m, formula (B.2) shows that the ΔCoVaR does not react on the shape of h at all, provided it is strictly increasing. Adding any monotonicity-preserving downward bias to the risk profile, for instance by additional put options held short, is neglected by the ΔCoVaR.

Now let us add idiosyncratic risk. If it is not too large, we can approximate the CoVaR by linearizing h in the tail. The CoVaR would be the ideal point of a Taylor expansion but is yet unknown, of course. We take $Q_\alpha\left(R_m\right)$ instead, the CoVaR in the absence of idiosyncratic risk. This gives

$$R_i \approx h\left(Q_a\left(R_m\right)\right) + h'\left(Q_a\left(R_m\right)\right)\left(R_m - Q_a\left(R_m\right)\right) + \varepsilon_i \equiv a_\alpha + b_\alpha R_m + \varepsilon_i$$

with $a_\alpha \equiv h\left(Q_a\left(R_m\right)\right) - h'\left(Q_a\left(R_m\right)\right)Q_a\left(R_m\right)$ and $b_\alpha \equiv h'\left(Q_a\left(R_m\right)\right)$. Recalling that R_m and ε_i are independent, the linearization also provides us with an approximation for the quantile of R_i:

$$Q_\alpha\left(R_i\right) \approx a_\alpha + \sigma\left(R_i\right)\Phi^{-1}\left(\alpha\right) = a_\alpha + \sqrt{b_\alpha^2\sigma_m^2 + \sigma_i^2}\,\Phi^{-1}\left(\alpha\right). \tag{B.3}$$

To condition on R_i, we need an orthogonal representation

$$R_m \approx c_\alpha R_i + d_\alpha + \nu \tag{B.4}$$

that fulfills $\operatorname{cov}\left(R_i, \nu\right) = 0$ and $\mathbf{E}\left(\nu\right) = 0$. Resolving (B.4) to ν and putting it into the covariance condition gives

$$c_\alpha = \frac{b_\alpha \sigma_m^2}{\sigma^2\left(R_i\right)}$$

and, combining this with the expectations of R_m and ν,

$$d_\alpha = E\left(R_m\right) - E\left(c_\alpha R_i\right) = \left(1 - c_\alpha b_\alpha\right)\mu - c_\alpha a_\alpha,$$

where μ is the expectation of R_m. As $c_\alpha R_i$ and ν must be orthogonal, their variances add up to the one of R_m, implying

$$\sigma_\nu^2 = \sigma_m^2 - c_\alpha^2 \operatorname{var}(R_i) = \frac{\sigma_m^2 \sigma_i^2}{\sigma^2(R_i)}.$$

As R_m is now represented as an approximate sum of two orthogonal components ν and R_i, the former is unaffected when we condition on the latter. Using (B.3), we obtain

$$\begin{aligned} Q_\alpha\left(R_m \middle| R_i = Q_\alpha(R_i)\right) &\approx c_\alpha Q_\alpha(R_i) + d_\alpha + Q_\alpha(\nu) \approx c_\alpha\left(a_\alpha + \sigma(R_i)\Phi^{-1}(\alpha)\right) + d_\alpha + \sigma_\nu \Phi^{-1}(\alpha) \\ &= \left[c_\alpha \sigma(R_i) + \sigma_\nu\right]\Phi^{-1}(\alpha) + c_\alpha a_\alpha + d_\alpha \\ &= \left[b_\alpha \sigma_m + \sigma_i\right]\frac{\sigma_m}{\sigma(R_i)}\Phi^{-1}(\alpha) + \mu\frac{\sigma_i^2}{\sigma^2(R_i)} \end{aligned}$$

and, with $g_\alpha \equiv b_\alpha \sigma_m / \sigma_i$,

$$Q_\alpha\left(R_m \middle| R_i = Q_\alpha(R_i)\right) \approx -\frac{g_\alpha + 1}{\sqrt{g_\alpha^2 + 1}}\sigma_m \Phi^{-1}(1-\alpha) + \frac{\mu}{g_\alpha^2 + 1}. \qquad \text{(B.5)}$$

Given typical relative sizes of volatilities and drift, the second term is very small compared to the first.[8] If b_α grows (and so g_α proportionally) while market and idiosyncratic risk remain fixed, the first (negative) term decreases for $g_\alpha < 1$, increases for larger values, and converges from below to $-\sigma_m \Phi^{-1}(1-\alpha)$. This means, increasing the portfolio's market sensitivity in the tail – be it by raising β_i or by increasing an option position with positive delta – can make the magnitude of the CoVaR *shrink*.

An important observation is that the approximate CoVaR does not depend on a_α, which is the intercept of the linearization of h. The invariance to the intercept has an interesting consequence. Assume the bank holds a *digital* option short that pays if the market falls below some threshold. Given also a short time to maturity, the option delta is nearly flat outside a small range around the trigger point. If the probability that the option pays is remote enough from the CoVaR's confidence level (above or below), the steepness b_α of the risk profile relevant for the CoVaR would remain nearly unaffected by the existence – and the size – of the short digital option position. The option does have impact on a_α, but that has a negligible one on CoVaR.

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8 Recall that we deal with a return horizon from 1 to 5 days. For daily returns, annual volatilities of the order of 20% transform into standard deviations of $20\% / \sqrt{260} = 1.2\%$, whereas drifts of 10% transform into $10\% / 260 = 0.038\%$.

In order to calculate the ΔCoVaR, which requires the CoVaR under R_i being at its median, another linearization can be performed with the median of R_m as expansion point. The linear approximation has then a steepness of $b_{0.5}$ (also defining $g_{0.5} \equiv b_{0.5}\sigma_m/\sigma_i$), which finally gives

$$\begin{aligned}\Delta CoVaR &= Q_\alpha\left(R_m \middle| R_i = Q_\alpha\left(R_i\right)\right) - Q_\alpha\left(R_m \middle| R_i = Q_{0.5}\left(R_i\right)\right)\\ &\approx -\sigma_m \Phi^{-1}\left(1-\alpha\right)\left[\frac{g_\alpha + 1}{\sqrt{g_\alpha^2+1}} - \frac{1}{\sqrt{g_{0.5}{}^2+1}}\right]\end{aligned} \qquad \text{(B.6)}$$

where the drift terms were ignored for their relatively small extent.

Let us now look at the particular case of the B portfolios. Their function h (represented by the solid line in the right upper panel of Figure) is increasing and concave, and hence steeper in the left-hand tail of R_m than at its median. The more options are proportionally added in the transition from portfolio B1 to B16, the larger b_α becomes (for instance, $b_{0.01}$ grows from 1 to 2.45), and the smaller becomes $b_{0.5}$ (it shrinks from 1 to 0.69). Both CoVaRs in (B.6) show the wrong trend; the ΔCoVaR does so accordingly.

In the case of portfolio type A with linear risk profiles, equation (B.6) is precise, with a uniform $b_\alpha = \beta_i$, and we obtain

$$\Delta CoVaR \approx -\sigma_m \Phi^{-1}\left(1-\alpha\right)\frac{g_\alpha}{\sqrt{g_\alpha^2+1}} = -\sigma_m \Phi^{-1}\left(1-\alpha\right)\left(\frac{\sigma_i^2}{b_\alpha^2\sigma_m^2}+1\right)^{-1/2},$$

with effects as analyzed in Section 3: while increasing the portfolio's beta is now accounted for by ΔCoVaR becoming more negative, i.e., in the correct way, increasing the idiosyncratic risk makes the absolute value of the ΔCoVaR shrink, ceteris paribus.

C Extrapolating systemic risk measures over confidence levels

In this appendix we provide an example of an extrapolation from MES and exposure CoVaR for higher confidence levels to the 0.1% level. We use the setup of Section 5.3 and examine exact risk measures evaluated at confidence levels 1%, 5%, and 10% as nodes. These values on the x-axis are represented on a logarithmic scale. Afterwards, a parable is laid through the three nodes and evaluated at $\log(0.1\%)$. This is the extrapolation value of the 0.1% risk measure.

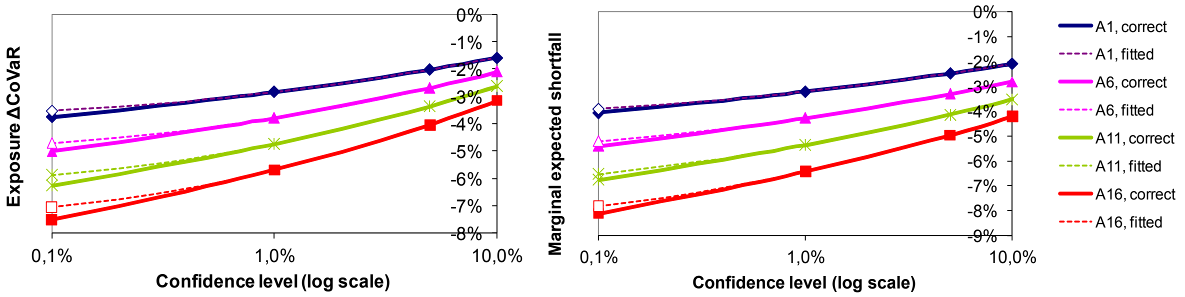
In Figure A1, solid lines represent correct MES or CoVaR under varying confidence levels. Dashed lines represent the fitted parables which coincide with their correct counterparts at those points in the right-hand half of each graph which are marked by filled symbols. Extrapolation values are marked by blank symbols.

While this extrapolation method performs well in setting A to C – in particular, it generates the right risk ranking within each setting – the last setting D sharply contrasts with the others. As in the main part of the analysis, a reversed risk ranking is suggested.

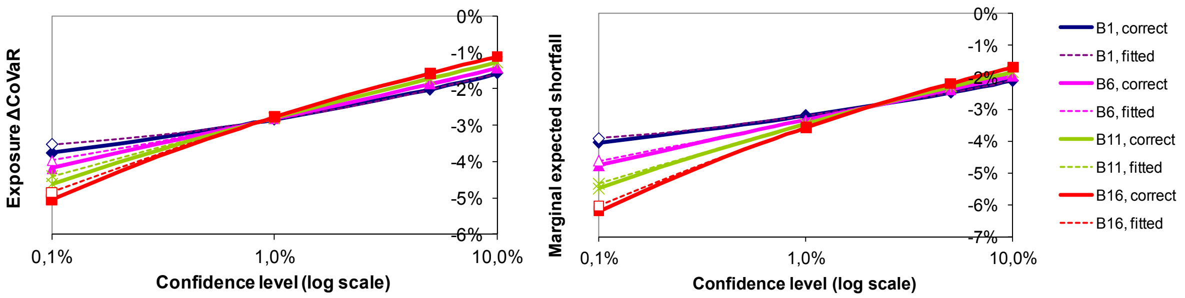
Figure A1: Extrapolating exact risk measures from moderate to extreme confidence levels

Confidence levels of 1%, 5%, and 10% build grid points on a logarithmic scale. Solid lines represent correct MES and exposure ΔCoVaR under varying confidence levels. The risk measure on 0.1% level is approximated by an extrapolating parable. Dashed lines represent the fitted parables which coincide with their correct counterparts at those points in the right-hand half of each graph which are marked by filled symbols. Extrapolation values are marked by blank symbols.

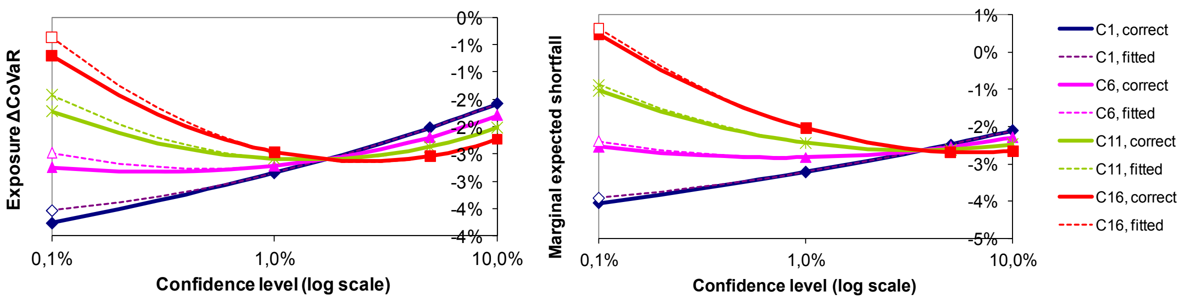
A portfolios



B portfolios



C portfolios



D portfolios

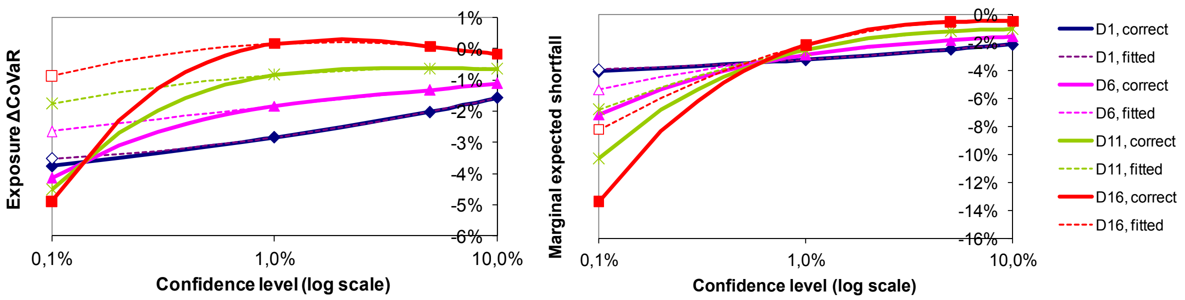


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Table 1: Simulated systemic risk measures in the presence of contagion

We simulate banking systems with N equally-sized banks. The banking system return is the value-weighted average of bank returns. Bank returns are driven by a common factor, idiosyncratic risk, and spillover from bank 1 to other banks:

$$R_1 = \beta_1 F + \varepsilon_1, \quad \text{(infectious bank)}$$

$$R_j = \beta_j F + \varepsilon_j + \gamma I_{\{\varepsilon_1 < \kappa\}} \varepsilon_1, \quad j = 2, ..., N \quad \text{(infected bank)}$$

$$R_S = N^{-1} \sum_{i=1}^{N} R_i$$

Parameters are set to $N = 50$, $\beta_1 = \beta_j = 1$, and (stated in per annum values): $E(F) = 0.05$, $\sigma(F) = 0.2$, $\sigma(\varepsilon_1) = 0.2$, $\sigma(\varepsilon_j) = \left(\sigma^2(\varepsilon_1) - \gamma^2 \operatorname{var}\left(I_{\{\varepsilon_1 < \kappa\}} \varepsilon_1\right)\right)^{1/2}$ for all $j > 1$. CoVaR measures are estimated through Monte Carlo simulation with 50 million trials. For $\Delta CoVaR_q$, they are computed with observations between the (q – 0.2%) and (q + 0.2%) quantiles of the conditioning variable. For the tail risk gamma, put prices are obtained through a separate Monte Carlo simulation, and then used in a regression of the bank's return on the system return and the change in the option price. Fields are shaded gray if the measure assigns more systemic risk to the infected than to the infectious bank.

	$\Delta CoVaR_{0.01}^{S\|R_i}$	$\Delta CoVaR_{0.01}^{R_i\|S}$	MES	Tail risk gamma
Panel A: γ = 0.75 κ = –0.0204				
Infectious	–2.091%	–3.154%	–3.103%	0.670%
Infected	–2.793%	–3.324%	–2.944%	–0.017%
Panel B: γ = 0.75 κ = –0.0289				
Infectious	–2.091%	–3.003%	–2.772%	0.217%
Infected	–2.581%	–3.091%	–2.705%	–0.005%
Panel C: γ = 0.75 κ = –0.0383				
Infectious	–2.090%	–2.930%	–2.602%	0.047%
Infected	–2.232%	–2.944%	–2.591%	–0.001%
Panel D: γ = 0.25 κ = –0.0204				
Infectious	–2.086%	–3.398%	–2.740%	0.201%
Infected	–2.139%	–2.937%	–2.618%	–0.005%
Panel E: γ = 0.25 κ = –0.0289				
Infectious	–2.090%	–3.082%	–2.630%	0.081%
Infected	–2.116%	–2.938%	–2.582%	–0.001%
Panel F: γ = 0.25 κ = –0.0383				
Infectious	–2.082%	–3.025%	–2.575%	0.016%
Infected	–2.092%	–2.903%	–2.567%	–0.001%

Figure 1: How ΔCoVaR responds to idiosyncratic risk

We examine a banking system with N equally-sized banks. Returns are described through:

$$R_i = \beta_i F + \varepsilon_i, \quad R_S = N^{-1} \sum_{i=1}^{N} R_i \text{ with } F \sim N(\mu, \sigma^2(F)), \quad \varepsilon_i \sim N(0, \sigma^2(\varepsilon_i)).$$

All ε_i and F are independent. For the analysis, banks are assumed to have uniform beta of 1; N–1 banks have a per annum idiosyncratic volatility of 0.2, while one bank has an idiosyncratic volatility of 0.4. The figure shows how $\Delta CoVaR_{\alpha}^{system,j}$ differs between the riskier bank and the other banks, depending on the number of banks in the system.

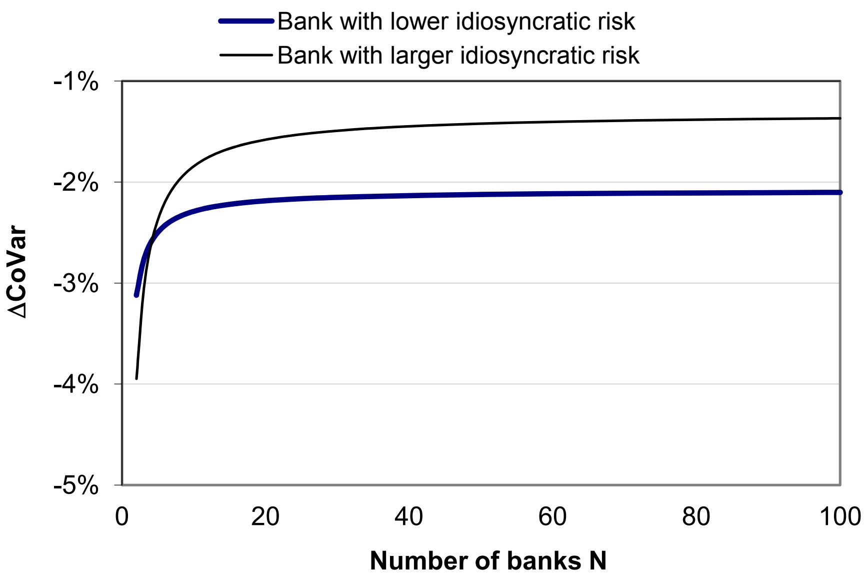


Figure 2: Simulated system returns versus returns of infectious and infected banks

We simulate a banking system with N equally-sized banks. The banking system return is the value-weighted average of bank returns. Bank returns are driven by a common factor, idiosyncratic risk, and spillover from bank 1 to other banks:

$$R_1 = \beta_1 F + \varepsilon_1 \text{ (infectious bank)},\quad R_j = \beta_j F + \varepsilon_j + \gamma I_{\{\varepsilon_1<\kappa\}}\varepsilon_1,\quad j = 2,\ldots,N \quad \text{(infected bank)},\quad R_S = N^{-1}\sum_{i=1}^{N} R_i$$

Parameters (per annum) are set to $E(F)=0.05$, $\sigma(F)=0.2$, $\sigma(\varepsilon_1)=0.2$, $\sigma(\varepsilon_i)=\left(\sigma^2(\varepsilon_1)-\gamma^2\,\mathrm{var}\left(I_{\{\varepsilon_1<\kappa\}}\varepsilon_1\right)\right)^{0.5}$ for all $j > 1$, $N = 50$. Panel A plots the full sample. Panel B contains only cases of contagion, where $\varepsilon_1 < \kappa$. Panel C contains cases of no contagion.

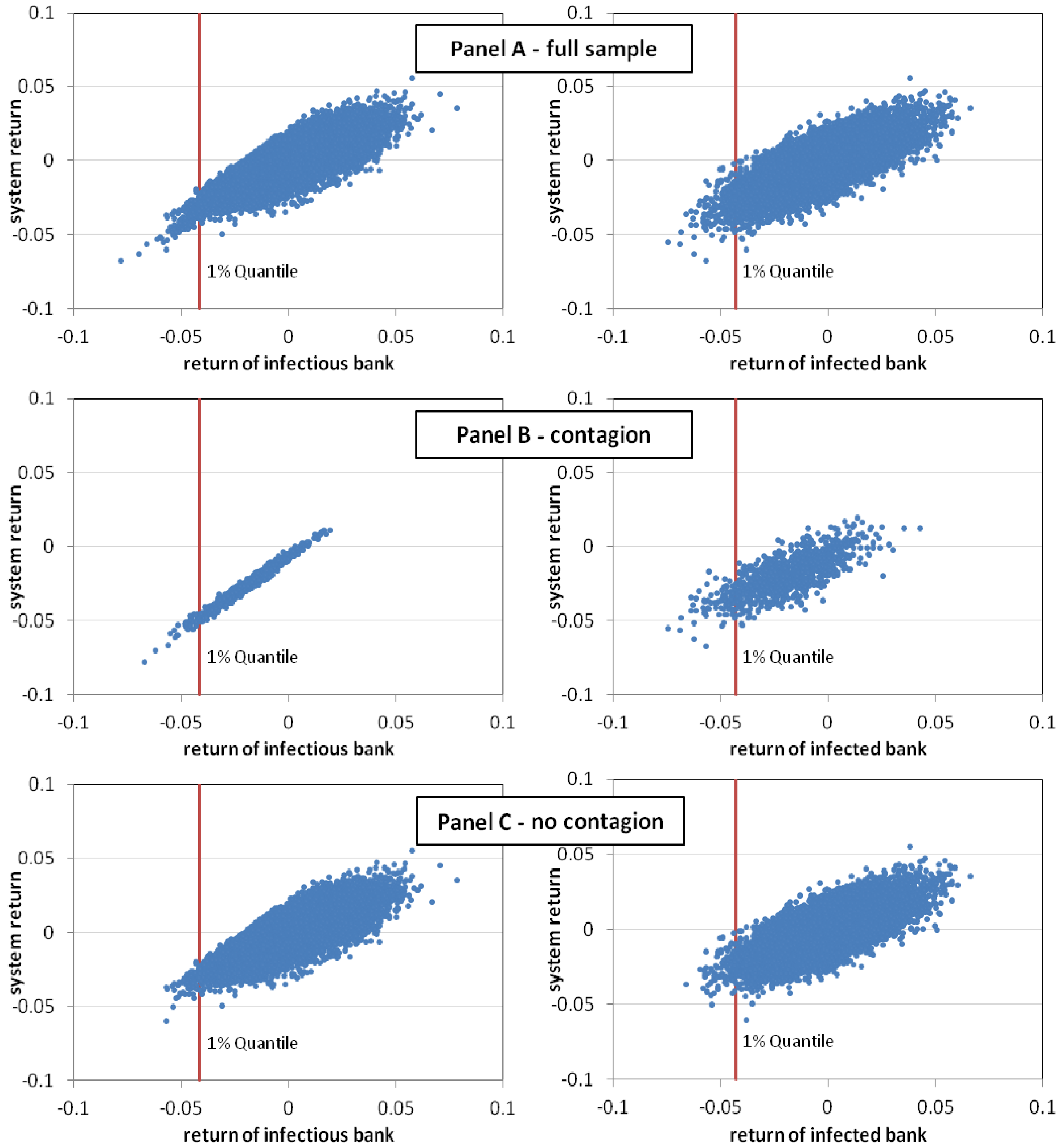


Figure 3: Market return versus portfolio return in four archetypical settings

Assuming that market returns follow a lognormal distribution with p.a. drift 0.05 and volatility 20%, we analyze daily returns of 16 portfolios in four settings, A to D. Portfolios No 16 are defined as follows:

A16: $\beta = 2$. No options.

B16: $\beta = 1$; –0.45% in one-month put with strike 0.8; 3% in one-month put with strike 1.

C16: $\beta = 2.125$; 0.75% in one-month put with strike 0.8

D16: $\beta = 1.375$; –4.5% in one-month put with strike 0.7; 5.7% in one-month put with strike 0.725

Weights for portfolios x.1 to x.16 obtain by linearly adjusting betas to 1, and option weights to zero. The idiosyncratic volatility is set to a uniform value of 20%. Option values are determined with Black-Scholes.

The solid line in each graph plots the market return against the expected portfolio return, conditional on the market return. The straight gray line marks the conditional return of an options-free linear portfolio with beta = 1.

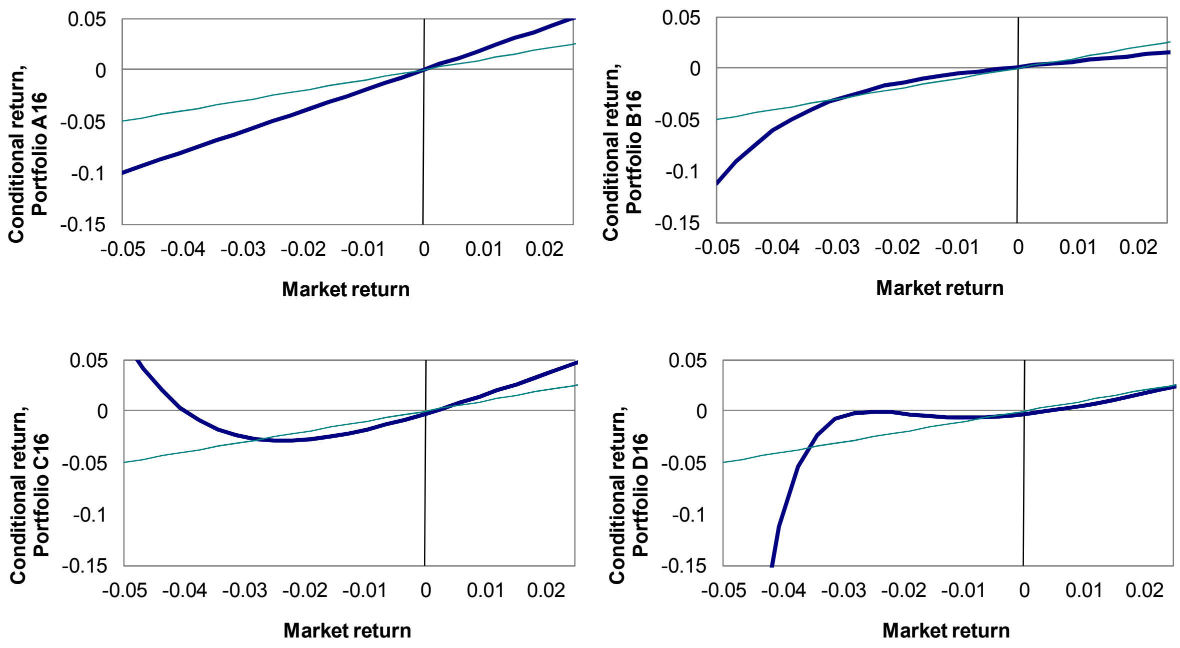


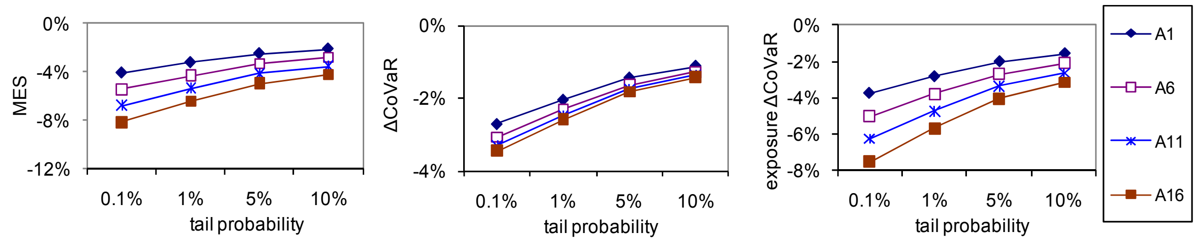
Figure 4: Exact systemic risk measures for different confidence levels

Assuming that market returns follow a lognormal distribution with p.a. drift 0.05 and volatility 20%, we analyze daily returns of 16 portfolios in four settings, A to D. Portfolios No 16 are defined as follows:

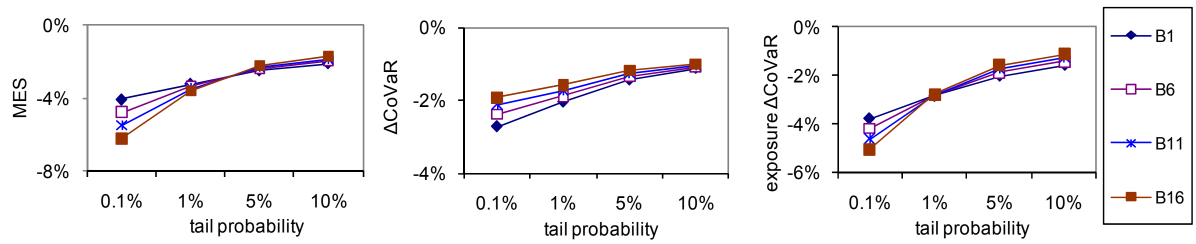
A16: $\beta = 2$. No options.
B16: $\beta = 1$; –0.45% in one-month put with strike 0.8; 3% in one-month put with strike 1.
C16: $\beta = 2.125$; 0.75% in one-month put with strike 0.8
D16: $\beta = 1.375$; –4.5% in one-month put with strike 0.7; 5.7% in one-month put with strike 0.725

Weights for portfolios x.1 to x.16 obtain by linearly adjusting betas to 1, and option weights to zero. The idiosyncratic volatility is set to a uniform value of 20%. Option values are determined with Black-Scholes. Marginal expected shortfall (MES), ΔCoVaR, and exposure ΔCoVar are calculated for tail probabilities of 0.1%, 1%, 5%, and 10% by analytical and numerical means.

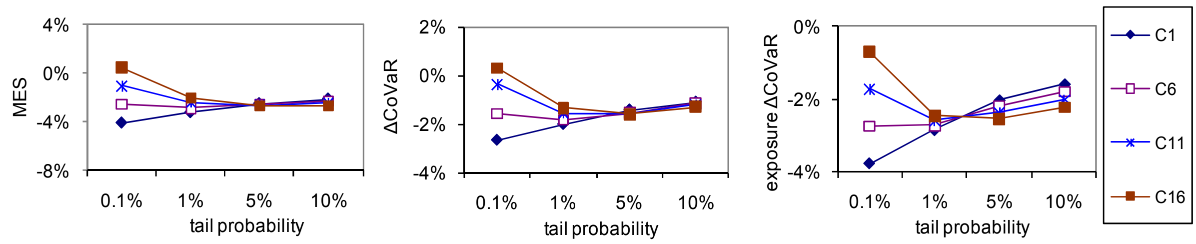
A Portfolios



B Portfolios



C Portfolios



D Portfolios

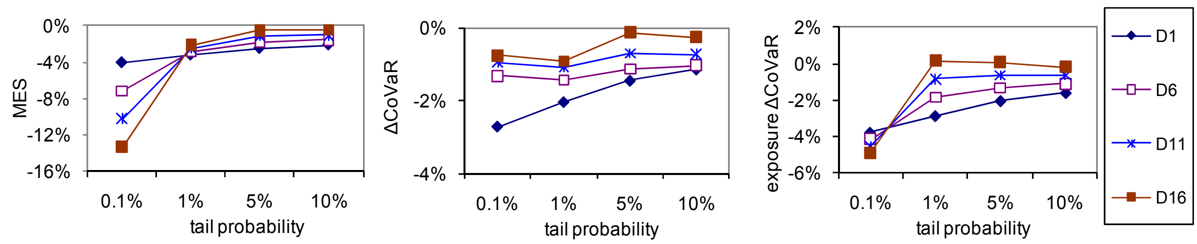


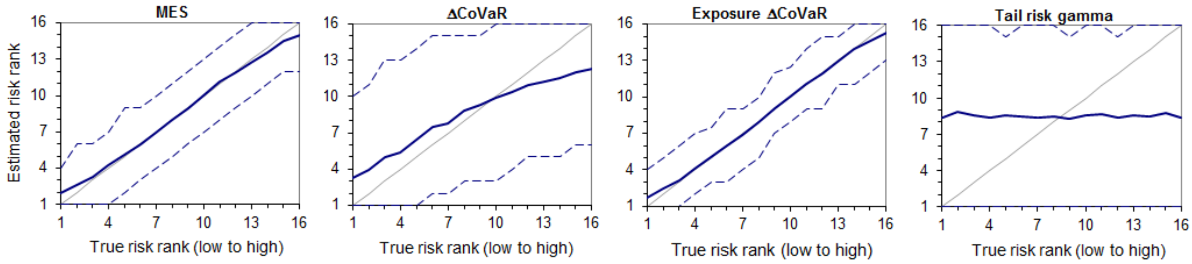
Figure 5: Simulated performance of systemic risk measures

Assuming that market returns follow a lognormal distribution with p.a. drift 0.05 and volatility 20% we simulate daily returns of 16 portfolios in four settings, A to D. Portfolios 16 are defined as follows:

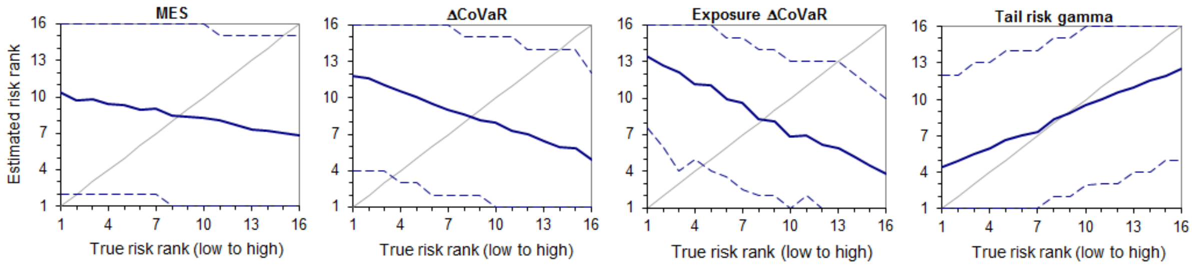
A16: $\beta = 2$. No options.
B16: $\beta = 1$; –0.45% in one-month put with strike 0.8; 3% in one-month put with strike 1.
C16: $\beta = 2.125$; 0.75% in one-month put with strike 0.8
D16: $\beta = 1.375$; –4.5% in one-month put with strike 0.7; 5.7% in one-month put with strike 0.725

Weights for portfolios x.1 to x.16 obtain by linearly adjusting betas to 1, and option weights to zero. Idiosyncratic volatility is set to a uniform value of 20%. Option values are determined with Black-Scholes. Marginal expected shortfall (MES, 5%) and tail risk gammas are estimated with 260 daily returns; ΔCoVaR and exposure ΔCoVaR (1%) with 1300 weekly returns. Simulation and estimation are repeated 1,000 times. The figures plot average estimated risk ranks, along with 90% confidence intervals, against the true risk rank as measured by MES (0.1%) and exposure ΔCoVar (0.1%).

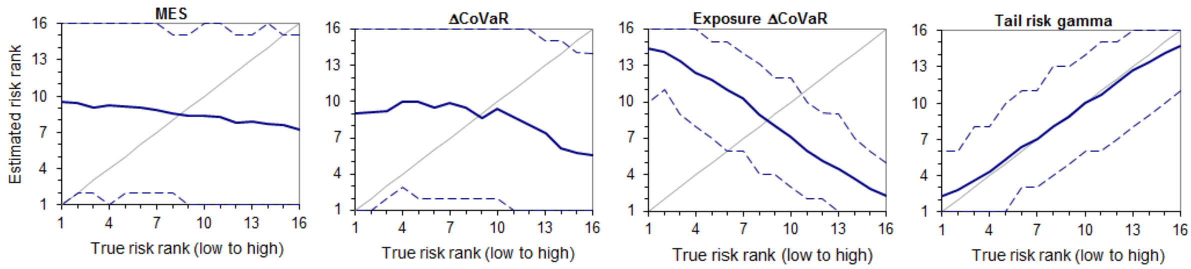
A Portfolios



B Portfolios



C Portfolios



D Portfolios