Systematic and Systemic Stress Tests

Martin Summer    Thomas Breuer
Oesterreichische Nationalbank
Fachhochschule Vorarlberg

Stress testing of banks is a form of economic and financial scenario analysis with one key question:

Which plausible scenarios lead to losses that are able to substantially impair a bank’s business?

An answer requires identifying dangerous scenarios by evaluating potential losses in a systematic way.
Introduction

Current stress tests have some major shortcomings:

1. They **neglect severe but plausible scenarios** and potentially create an illusion of safety.
2. They **consider scenarios without regard to their plausibility** thereby dealing with potentially highly implausible scenarios.
3. They **neglect systemic risk** and thereby underestimate the magnitude of potential losses.

Our paper offers ideas to address all of these problems with new methods and within a new framework.
Our paper is related to the literature in

- quantitative risk management
- coherent risk measures
- fire sale modelling
- reverse stress testing

Our paper is also related to concepts of modeling decisions under ambiguity.
Stress Tests: Key concepts

A stress test is based on a model of future value changes of a given financial portfolio with a simple intertemporal structure of “today” and “tomorrow” and with three main concepts:

1. Risk factors
2. Scenarios
3. Portfolio valuation functions
Scenarios in traditional stress testing

Traditional stress testing thinks of scenarios as *realizations* from a *multivariate distribution* of risk factors. While intuitive this is often *problematic* for a number of reasons:

1. Sometimes forces modellers to distinguish market risk and credit risk sharply.
2. Ignores that even for marketable positions it is unclear at which price exactly they can be liquidated.
3. Ignores that in distress risk factor distributions themselves change.
Generalized scenarios

We propose to think of scenarios as distributions of risk factors rather than as realisations. Given a fixed future time horizon, let $\mathbb{P}_0$ a risk factor distribution estimated from historical data. The future value of a position $X$ is then given by

$$V(\mathbb{P}_0) = E_{\mathbb{P}_0}(X),$$

A scenario is an alternative distribution $\mathbb{Q}$ for which the value of the position becomes

$$V(\mathbb{Q}) = E_{\mathbb{Q}}(X)$$
Systematic generalized scenarios

We call a stress test **systematic** if it provides a procedure to quantify the **plausibility** of a scenario and if it considers a **complete set** of scenarios, meaning **all scenarios at or above a certain threshold of plausibility**.
Measuring plausibility by relative entropy

We measure the **plausibility** of a generalized scenario $Q$ by its **relative entropy** with respect to some reference distribution $P_0$. The relative entropy of a probability distributions $Q$ with respect to a reference distribution $P_0$ is defined as

$$D(Q||P_0) := \begin{cases} \int \log \frac{dQ}{dP_0}(r)Q(dr) & \text{if } Q \ll P_0 \\ +\infty & \text{if } Q \not\ll P_0 \end{cases}$$

where $Q \ll P_0$ denotes absolute continuity of the distribution $Q$ with respect to the distribution $P_0$. 
Systematic Stress Test

The **systematic stress test procedure** searches for the worst expected value of the portfolio valuation function among the sufficiently plausible generalised scenarios:

$$\inf_{Q: D(Q||P_0) \leq k} E_Q(X).$$

The solution to this problem is a new distribution $\overline{Q}$, which we call the **worst case distribution**.
An illustration of the optimization problem

\[ D(Q||!) \leq k \]

all distributions \( \ll \) \( \nu \)

model class
Including systemic risk

Typically in financial distress losses get amplified by the interaction of bank behavior and the pricing of risk.

A frequent case, which we want to include for our loss evaluation model, is fire selling of marketable assets.

More complicated because it requires the inclusion of assumptions about bank behavior.
Price impact modeling

We combine our stress test with a recent price impact model from the literature due to Cont and Schanning 2017.

The behavioral assumption is that banks who have to take losses that lead to a leverage beyond a certain threshold start to sell marketable assets proportionally to restore a sufficient capitalization.

This causes an indirect impact on all other banks holding these marketable assets on their balance sheet because they have now to be revalued and are marked to market.
Price impact modeling

• The literature on price impact assumes that every asset class has an impact function $\Psi$ which maps liquidation values $q$ into relative price changes in this asset class.

• It is assumed that $\Psi$ is increasing, concave and satisfies $\Psi(0) = 0$.

• A common specification is a linear impact function of the form

$$\Psi(q) = \frac{q}{D} \quad \text{with} \quad D = c \frac{ADV}{\sigma}$$

• When an institution has to liquidate and amount $q$ of a particular asset with a current price $S$, this will depress the asset price to $S' = S(1 - \Psi(q))$
An example using public data

• We use exposure data from the EBA 2016 stress testing exercise 51 European banks with a balance sheet bigger than 30 billion Euro in 2016.

• Rather than trying a fully fledged stress test, we use a very reduced risk model for illustrative purposes: We assume that there are only two risk factors, residential mortgages in Italy and in Spain where the risk factors are house price indices for these countries published by the BIS.

• We use a threshold credit risk model like in our example 2 to find a worst case distribution of risk factors.

• Evaluating the losses from these credit exposures for the banking system as a whole we ask whether additional consideration of potential fire sales would amplify losses and if yes by how much.
## Portfolio of the banking system

**Example Bank (31.12.2015)**

<table>
<thead>
<tr>
<th>Assets</th>
<th>Liabilities</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Marketable assets</strong></td>
<td><strong>Debt</strong></td>
</tr>
<tr>
<td>€7270 Billion</td>
<td>€24951 Billion</td>
</tr>
<tr>
<td>- Corporate bonds AA</td>
<td></td>
</tr>
<tr>
<td>- Corporate bonds ZZ</td>
<td></td>
</tr>
<tr>
<td>- Sovereign bonds AA</td>
<td></td>
</tr>
<tr>
<td>- Sovereign bonds ZZ</td>
<td></td>
</tr>
<tr>
<td><strong>Non-marketable assets</strong></td>
<td>€19049 Billion</td>
</tr>
<tr>
<td>- Residential exposures AA</td>
<td></td>
</tr>
<tr>
<td>- Residential exposures ZZ</td>
<td></td>
</tr>
<tr>
<td>- Commercial exposures AA</td>
<td></td>
</tr>
<tr>
<td>- Commercial exposures ZZ</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>Total</strong></td>
</tr>
<tr>
<td>€26319 Billion</td>
<td>€26319 Billion</td>
</tr>
</tbody>
</table>
Distribution of Leverage across the 51 EBA banks
Distribution of marketable and non marketable assets
Pds residential mortgages Italy and Spain
Share of losses due to initial shock and due to deleveraging
Deleveraging and Loss amplification

![Graph showing the relationship between Price impact iterations and the number of banks selling assets. The graph peaks around price impact iteration 6, showing a significant increase in the number of banks selling assets.]
Price impact
Conclusion

• The methodology of current stress testing is problematic both in the way stress scenarios are chosen and in the way bank losses are evaluated. Scenario selection and systemic risk are issues that have been somewhat neglected.

• Our paper calls for changing the focus in stress testing on systematic scenario selection and on the consideration of loss amplification by systemic risk.

• The reason is that otherwise, stress tests will be weak in answering the key questions: “Which scenarios lead to big losses?” and “How big are the worst losses?”.
Conclusions

- Specifically we propose to work with generalised scenarios.
- We argue that it is useful to think about scenarios as distributions rather than as realisations. This allows for an integrated analysis of market and credit risk at a common time horizon.
- Systematic scenario selection is achieved by an appropriate form of worst case search over plausibility domains. This method finds among all equally plausible scenarios the scenario leading to the worst expected loss for any given portfolio.
Conclusions

- For making stress tests more **systemic**, we propose an integration of recent results on the **quantitative modelling of deleveraging processes** with our approach to systematic scenario search.

- We believe that our proposals can be implemented in a fairly straightforward way without drawing on exotic or new data sources. Our example gives ideas about how such an approach might work in a more or less traditional top down stress testing setup.