EBA STAFF PAPER SERIES

N. 17 – 09/2023

THE CALIBRATION OF THE IRB SUPERVISORY FORMULA – A CASE STUDY

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ABSTRACT

The level of capital requirement generated by the IRB approach depends crucially on the asset correlation, a parameter that enters the regulatory risk weight formula and is determined by the Regulators. Several studies have estimated the asset correlations and found that the empirical values are materially lower than the regulatory calibration included in the Basel framework. However, the simple comparison between different estimates of this parameter does not easily translates into a clear economic interpretation. In this paper, we use detailed data from Italian banks to show how to extract from the regulatory risk measures easily interpretable figures i.e. the Worst-Case Default Rate (WCDR) and the Worst-Case Loss (WCL) and we show how the asset correlation influences these measures. We then provide a rationale for the regulatory calibration in terms of corrections to well-known limits of the underlying models like the assumption of perfect granularity. We claim that our approach can provide a better understating of the IRB risk measures fostering their transparency and reliability but also simplifying the comparison among different banks. We apply the proposed approach exploiting some data sources (publicly available and proprietary). As the data used is mainly referred to the Italian system and, in particular, to only two banks, the empirical results obtained are meant just to provide a practical example.

KEYWORDS

Bank Capital; Regulation; Basel 2; Credit Risk; Asset Correlation; Value-at-Risk

JEL CODES

C15; G21; G32

1. Introduction[1]

Motivation

About 20 years ago, the Basel Committee on Banking Supervision (BCBS) introduced into the system of prudential regulation for banks a risk-based framework (named Basel II), allowing financial institutions to use internal models to calculate minimum capital requirements for major risk types. For credit risk, the BCBS proposed a measure of regulatory capital based on risk measures internally estimated by the banks, and presented the first version of the Supervisory Formula (SF) i.e. a “closed” formula aimed at replicating the results of the portfolio models developed by major investment banks and consulting firms (mainly from the US). The BCBS opted for a structural model and, in particular for the gaussian[2] Single Risk Factor Model derived from the Merton-Vasicek (MV) model. The banking regulation on credit risk issued by the BCBS has undergone a progressive evolution starting from the initial amendment proposal submitted for consultation in 1999[3], which proposed the use of systems based on internal rating models (IRB) for the purpose of calculating the capital requirement. Detailed clarifications were provided by the consultation paper published in January 2001[4].

The basic idea of the regulation consists in establishing the minimum amount of capital that the bank must hold to protect depositors (and the entire economic system) from the insolvency of the bank itself. The minimum capital has been quantified in different ways depending on the versions of the proposed regulation, but all share the principle of protecting the bank from adverse credit events (peak losses) that may affect the loan portfolio up to a certain confidence interval over the time horizon of one year. The worst-case credit loss of the portfolio obtainable from the MV model is based on the probability of default conditional on the realisation of an extreme event, which has a small but not zero probability of occurrence.

The level of capital requirement generated by the IRB approach depends crucially on the asset correlation, a parameter that enters the regulatory risk weight formula and is calibrated by the Regulator. Estimating the asset correlation parameter is challenging, however, according to Chernih (2006), there appears to be a growing consensus in the literature on the range for asset correlations and the current regulatory correlations are larger than the empirical results reported in this literature. However, differences in the estimating approaches and in the type of data, may produce quite different results. The simple comparison of different estimated values of this parameter does not provide an easy interpretation. For example, what it does entail in practice to assume an asset correlation equal to 1% or 20% is not self-evident.

In this paper, we offer a different perspective that allows us to evaluate the adequacy of the asset correlation estimate for a given portfolio without the need to make comparisons with other estimates, perhaps based on data that is not representative of the portfolio in question. We show that the IRB credit risk measure is substantially obtained by rescaling the distance between the expected probability of default (PD) and the stressed PD. The regulatory IRB Supervisory Formula[5] (SF hereinafter) can be seen as an algorithm whose main purpose is to provide the stressed PD given the expected PD (estimated by the banks) and the asset correlation. The stressed PD can then be compared with the observed default rates. This comparison provides a simple way to evaluate the realism (and the level of conservatism) implied by a given level of asset correlation. We then turn our attention to the minimum regulatory capital requirements. We show that this quantity can be seen as the

[1] The opinions expressed are those of the Authors and do not involve responsibility of the institutions.

[2] In practice, the external/systemic factor is assumed to be a standard normal random variable

[3] BCBS (1999a)

[4] BCBS (2001a). in preparation for this regulatory proposal, a survey of the state of the credit risk models available was carried out by the Basel Committee, BCBS (1999b).

[5] Article 153 and 154 of the Regulation EU 2013/575 (the CRR)

estimated worst-case loss (WCL) given the confidence level and how this measure is influenced by the stressed PD. This enables us to make comparisons with different risk measures obtained by relaxing some of the hypothesis underlying the IRB approach and this in turn permits to quantify the role of such hypothesis.

Structure of the paper

The rest of the paper is organised as follows: Section 2 provides a review of the literature regarding the estimation of the asset correlation. In Section 3, four different sources of data (two publicly available and two propriétaire data) are used to estimate the asset correlation. The data is mainly referred to the Italian system and in particular to two Italian banks. We find values for the asset correlations in line with the existing literature. In Section 4 we show the effect of asset correlation on the determination of the stressed PD. The latter is then compared with the worst-case level of default rates actually observed in a time frame which includes the Great Financial Crisis of 2007-2008. We argue that this comparison enables to appreciate, in a more natural way, the level of conservatism implicit in the regulatory calibration of the asset correlation. In Section 5 we turn the attention to the regulatory minimum capital requirements, and we show that the level of conservatism of the stressed PD can be seen as covering risk components that the Basel Committee avoided formalising for the sake of simplicity and due to lack of data. We empirically verify this hypothesis by processing the data of real portfolios of two Italian banks. In detail, we have analysed the difference between the measure of the worst-case loss provided by the SF that uses the regulatory asset correlation and the measure that would be obtained using the asset correlation estimated based on real default data but relaxing some of the assumptions underlying the IRB model. The results of this empirical exercise cannot be easily extended given the limited sample (two banks) at our disposal. However, our findings suggest that by expressing the regulatory risk measure in terms of WCL (which should be covered by provisions and capital), instead of unexpected loss multiplied by 12.5[6] and disentangling its implicit components, could greatly simplify the comprehension of these metrics by the market participants. Transparency and the possibility of making comparisons among banks would benefit.

2. Literature Review

The default correlation

In his book on credit-risk modelling, Bolder (2018) states that the dependency on default correlation is at the heart of credit-risk modelling. The imposition of even quite modest amounts of default dependence has dramatic effects on risk estimates. Loosely speaking, without the hypothesis that defaults are somehow correlated, the number of defaults generated by a portfolio of N borrowers could be represented with a binomial distribution[7] of parameters N and p where p is the probability that a single borrower incurs a default. In that case, the expected value of the default rate (number of defaults divided by N) would be p and the variance $p(1-p)/N$.

The fact that the variance decreases with N implies that, for N sufficiently large, the variance – and consequently the unexpected loss – would go to zero. In other words, assuming default independence, for sufficiently large portfolios, all idiosyncratic risks can be diversified completely. On the other hand, with even a limited amount of correlation among the defaults, a systematic, non-diversifiable source of risk is introduced. From a risk management perspective, a significant underestimation of the credit risk exposure can derive from omission

[6] This would also align the prudential treatment of the credit risk with that of the market risk.

[7] Let D_i represent the dichotomous variable assuming value 1 if the $i-esim$ borrower defaults and 0 otherwise. Then $\mathbb{E}(D_i)=p$ is the expected probability of default. Assume that in a portfolio there are N borrowers having the same probability of default. Then, under the hypothesis that the defaults are independent, the total number of defaults is the sum of N independent Bernoulli variables so $\sum_{i=1}^{N} D_i$ is a binomial random variable with parameters N and p.

(Das, 2006) or underestimation (Jorion, 2006) of the default correlation. The Annex presents a simple example built with real data showing the difference between the actually observed variance of a series of default rates and the theoretical variance obtained under the hypothesis of independence.

In credit risk modelling, the dependency on the default correlation is typically introduced in two different ways (Bluhm, 2016): reduced form and structural models. The aim of these models is to obtain the theoretical probability distribution of the defaults and to derive from it the risk measures. In reduced-form models, the defaults of the borrowers are seen as exogenous events occurring with a given probability and a given default correlation. In structural models, the defaults are made endogenous, in the sense that the model explains their occurrence through a mechanism and the risk measures are derived from the mechanism that generates the defaults. It is under this second type of models that the asset correlation plays a role.

A third approach, quite different from the reduced and structural models, is the econometric approach where we try to link the historical series of default rates to macroeconomic variables, such as the GDP. In these models, an effort is made to identify the external factors that determine the dynamics of the default rates, while the risk measure is obtained by stressing these external factors, for example by assuming a strong reduction in GDP.

The reduced form and the structural models and the econometric models share the goal of providing a measure of stressed default rate, but they arrive at this result in quite different ways. The reduced form and the structural models allow to construct the entire probability distribution of the default rates and to derive from this the stressed default rate (the WCDR) given the desired confidence level. With econometric models, on the other hand, the stressed default rate is obtained by providing a stressed scenario.

Under the structural framework, the default of a borrower is an event connected with an external factor, which is in common with all the borrowers of the portfolio. It is through this common dependence on the external factor that the defaults are correlated, while the strength of the dependence between the common factor and the probability of default is named asset correlation. In other words, the hypothesis of default independence is substituted by the hypothesis of correlated defaults and in turn this correlation stems from the connection between the probability of default and a factor in common with all the borrowers that can be thought of as the state of the economy.

Stock prices or default rates

The asset correlation can be estimated either directly, by exploiting equity prices, or indirectly (passing through the default correlation), by exploiting data on default rates. Using default data, one can estimate default correlations directly and then retrieve the asset correlation. Using asset value data, one can directly estimate asset correlations and then obtain the default correlations.

Using equity prices appears more appealing for several reasons: the original derivation of the Merton model refers to companies whose equities are traded on the financial markets; data about equity prices is more easily found while banks' default data is not public, and the time series is usually longer[8]. However, it must also be considered that equity prices are usually referred only to large corporates while banks' portfolios include a substantial number of SMEs[9] and exposures towards households. In general, market-based data can be rationally used whenever there is a liquid and reasonably efficient secondary market.

[8] The Basel 2 default definition has been implemented by the banks since 2006, which implies that the longest time series available with a homogeneous definition of default spans 15 years.

[9] From Piccone (2019) pag 78, the share of corporates with more than 250 employees between the non-financial firms is equal to 0.47% in Germany, 0.14% in France and 0.09% in Italy while the share of enterprises with less than 10 employees is 82% in Germany, 95% in France and in Italy.

The main difference between estimates obtained through default data and market data is the sample composition. Market data is available only for publicly traded enterprises, and these are usually large corporates. With default data it is possible to estimate the default correlation (and derive the asset correlation) for any sector including SMEs and households. Asset correlation estimates based on market data tend to be larger than those based on default data. The higher estimated asset correlation stemming from market data could be led by the fact that large corporates are more subject to the financial markets where the pressures of market participants, herd behaviour and the tendency to anticipate the effects of shocks, may lead to a higher dependency from common factors. These aspects may lead to the question of whether it is fair to derive from market data-based estimates the measure of a crucial parameter, such as the asset correlation for sectors where the idiosyncratic components can be expected to be structurally (at least, in the short term) more important than the external conditions.

Duellmann (2010) investigates why estimates of asset correlations based on equity prices tend to be considerably higher than those based on default rates. It is shown that differences between these estimates are caused by a substantial downward bias of the estimates based on default rates. More specifically, by means of a simulation study, it is shown that the direct estimation of asset correlations from equity returns is superior to an estimation from default rates, both in terms of bias and efficiency. The problem of the downward bias in the estimation of the asset correlation based on default rates, has been discussed also by Gordy (2002) and Resti (2008).

From a prudential perspective, the possibility that the asset correlation may not be stable over different economic scenarios is crucial. In Lee (2011) it is found that the asset correlations are asymmetric and have a procyclical dynamic: they tend to rise during economic downturns but decline during economic upturns. Working with stock prices could enable to better explore these dynamics. However, the relationship between the probabilities of default (implicit in the CDS) and the actual default rates must be further investigated.

Asset correlations: estimates vs regulatory

For Italian banks, Sironi (2003) obtained estimated asset correlations consistently lower than the value proposed by the BCBS; exploiting publicly available data stemming from the Italian Credit Register, Resti (2008) estimated for the non-financial sector (Corporates and SMEs) a value equal to 2.4%; Curcio (2011) estimated the asset correlations for firms classified in different buckets of size and area obtaining values that are in the range of 0.5% and 5%. For German banks Duellmann (2013), using a dataset including about 250 thousand of borrowers per year (SMEs and Large Corporates), obtained asset correlations not higher than 2%. Henneke (2006), by examining a typical German SME portfolio of Small and Mid-Sized Enterprises, also found asset correlations to be lower than those assumed in the capital accord. In a study of the Bank of Japan, Hashimoto (2009), the estimates referred to large and medium-sized corporates, SMEs and households and to different industrial sectors, showcased that asset correlations never exceeded the value of 4.5%. In his survey, Frye (2008) reports the results in Table 1 below. It can be noticed that the asset correlations obtained from default data, as in our case, are systematically lower than the ones obtained from asset value data.

Table 1: Estimated asset correlations reported in Frye (2008)

Source Study	Data Source	Results
Gordy (2002)	S&P	1.5% - 12.5%
Cespedes (2000)	Moody's	10%
Hamerle *et al.* (2003a)		max of 2.3%
Hamerle *et al.* (2003b)	S&P 1982 - 1999	0.4% - 6.04%
Frey *et al.* (2001)	UBS	2.6%, 3.8%, 9.21%
Frey & McNeil (2003)	S&P 1981 - 2000	3.4% - 6.4%
Dietsch & Petey (2004)	Coface 1994 - 2001	0.12% - 10.72%
	AK 1997 - 2001	
Jobst & de Servigny (2004)	S&P 1981 - 2003	intra 14.6%, inter 4.7%
Duellmann & Scheule (2003)	DB 1987 - 2000	0.5% - 6.4%
Jakubik (2006)	BF 1988 - 2003	5.7%

Asset correlations from default data

Source Study	Data Source	Results
Duellmann *et al.* (2006)	KMV	10.1%
KMV (2001)	Undisclosed	9.46% - 19.98%
Fitch (2005)	Equity	intra 24.09%, inter 20.92%
Lopez (2002)	KMV Software	11.25%

Asset correlations from asset value data

As regards, it is also interesting to mention the results obtained by Moody's/KMW researchers. In detail, in Zhang (2008) the estimated values are in line with the Bcbs proposals, the data used are referred to publicly traded U.S. non-financial firms, in practice U.S. Large Corporates. Chernih (2010), by summarizing different studies using rating and default data from rating agencies suggested the asset correlation parameter for the corporate sector to be around 10%. Siarka (2014) exploiting data obtained from financial institutions operating in Poland obtains for Retail loans estimates of the asset correlation in the range 1% – 3%. The author concludes that: << […] *it appears that the correlation between retail borrowers' assets is very low, much lower than that recommended under the IRB approach*>>. Geidosch (2014) uses several estimation approaches to obtain an estimate of the asset correlation for US residential mortgage-backed security transactions, and the result is about 6%. Duellmann (2014) highlights that a possible underestimation of the asset correlation could result from the limited length of the time series. However, in that paper, the estimates are obtained with data covering 7 years and not including a complete business cycle. Dietsch (2016), by using a unique and comprehensive data set related to France and Germany capturing a significant part of lending towards SME and large corporates, obtained estimates of the asset correlation in the range of 0.5% - 2%. In a recent paper, Di Clemente (2020), by exploiting one of the data sources that we used for this paper (i.e. the historical time series of default rates for Italy that can be freely downloaded from the website of the Bank of Italy) estimated asset correlations in the range of 1%-3%.

In Resti (2008) it is studied the effect on the estimation of the asset correlation of working with default rates stemming from samples of borrowers that are not homogeneous in terms of probability of default and he concludes that the effect is a downward bias. These considerations are in line with those in Gordy (2002) about the possible sources of bias in the estimation of the asset correlation. For example, Blumke (2017), while stressing the importance of working with homogeneous portfolios, analyzed Standard and Poor's rating and default data concluding that asset correlation parameters for banks and several industry sectors, are close to the regulatory value or even exceed it.

However, it must be noticed that data exploit in this paper spans at least 16 years (from 2007 to 2022), including the effect of the Great Financial Crisis. Moreover, we estimated the asset correlation at rating grade level and this should entail a good level of homogeneity. The item of the possible underestimation of the asset correlations is resumed further in this paper where a simple, heuristic solution is also suggested.

Lastly, it is worth mentioning the possibility that the asset correlation is not stable over the economic cycle. One of the Great Financial Crisis aftermath is that correlations between assets in periods of stress can differ substantially from those seen in normal circumstances. The problem of "correlation breakdown" during periods of greater volatility is well known. The subprime CDSs crisis was induced largely by badly calibrated mortgage default correlations. Also, with reference to 1998, when the market conditions were heavily stressed, a comprehensive study (BIS (1999)) found that the average correlation between yield spreads rose significantly during the crisis.

3. Estimating asset correlations

Data

In the following sections, we exploit four data sources. For all of them, the length of the time series ensures the full coverage of different economic cycles, including the effect of the 2008 financial shock and the consequent 2011-2012 European sovereign debt crisis. The magnitude of the 2008 crisis is clearly observable in the default rates (see Figure 2). All the data sources are referred to non-financial corporations.

The first data source is the time series of the default rates published by Moody's. The observation period dates to the 80s (so more than 40 years are covered) it is also available the distinction between Investment and not-Investment grades. The main con of this data source is that it refers mainly to US large enterprises.

The second data source is made available by the Bank of Italy, and it is based on the Italian Credit Register[10]. The period covered is 2006 – 2022 (17 years). The data refers to practically all Italian non-financial corporations. The default definition is based on the Regulatory definition of default (Article 178 Regulation (EU) 575/2013) that in turn is in line with the Basel 2/3 rules. This definition is wider than the insolvency legal criterion.

The third and fourth sources are not publicly available. Cerved Group S.p.A. is a group that operates as a commercial information agency: it assesses the solvency and creditworthiness of companies, monitors and manages credit risk and defines marketing strategies. Cerved operates as a rating agency through Cerved Rating Agency S.p.A. We exploit Cerved data referred to non-financial corporations segmented in rating grades. The main con, in this case is the definition of the default that, on the contrary for the other data sources, is not aligned with the Basel definition. In the Cerved database, the judiciary definition of default is used.

Lastly, we exploit the data referred to the regulatory asset class Corporates of two Italian multiregional banks whose internal credit risk models have been authorized for their use for regulatory purposes and are currently in place. This is the most granular and complete data that we use in this paper. Clearly, it is referred to just two particular banks and even if these two are quite differentiated by size and regional areas of main interest, they still do not constitute a representative sample of the underlying banking system. In detail, two different data sets were provided. The first data set (DS1) provides the time series of the yearly default rates observed for each rating grade between 01/01/2007-01/01/2022 of the banks' master scale and it is used to estimate the asset correlation. The second data set (DS2) contains data at the borrower level at a given (not specified) reference date. For each borrower, this data set contains the exposure at default, the estimated LGD (inclusive of the downturn effect), the rating grade and the associated probability of default and the RWA computed by the bank along with the SF. The DS2 is used to compute the risk measures exploiting the SF both with the regulatory and the estimated asset correlation. It is also used to obtain the risk measures through Monte Carlo simulation.

[10] https://www.bancaditalia.it/statistiche/raccolta-dati/centrale-rischi/index.html?com.dotmarketing.htmlpage.language=1

Table 2: Data Sources Used[11]

Source	time series		Ratings	Definition of default	Portfolio
	range	T			
Moody's	1981-2022	42	2 grades	Market	Traded Corporates
Bankit	2006-2022	17	No	Basel 2/3	Non Fin. Corps
Cerved	2008-2022	15	8 grades	Judiciary	Non Fin. Corps
Bank1	2007-2022	16	9 grades	Basel 2/3	Corporates
Bank2	2009-2022	14	15 grades	Basel 2/3	Corporates

Estimates

There exist several methods that can be exploited for the estimation of the asset correlation (R), such as the likelihood method which permits a direct estimation of the R parameter, or the method of moments, which provides an estimation of the default correlation, from which in turn the R is derived. We used eight different estimation approaches to estimate the asset correlation parameter. The estimation was done at the aggregate level for the first and second data sources, while for the third and fourth data sources, the estimation was done for each rating grade. The chapter 9 of Baesens (2016) provides details about all but one these estimation approaches together with SAS codes implementing them. The estimator ML2 is the same used in Dietsch (2016).

The results referred to one of the four data sources are shown in the table below. The probability of default (PD) associated with each rating grade was computed as the simple average of the annual default rates. In the second column the value of the R parameter obtained with the regulatory expression (7) is reported. The difference between the regulatory and the estimated value of the asset correlation is quite wide with all the estimators. In particular, for lower levels of the PD the difference is higher. For example, for the IG rating grade of Mood'y data, the regulatory asset correlation is about 23.5% against 12.7% obtained with one of the estimators.

Indeed, the Regulatory calibration is obtained under the assumption that lower PD levels are connected with higher levels of asset correlation. The effect of this assumption is evident in Figure 1 which represents the series of the regulatory and estimated asset correlation for the specific case of Bank2. The decreasing trend of R with respect to the PD is observed also in the estimated values but only up to a certain PD threshold beyond which R starts to grow again.

The values we obtain range between 0.3% and 12% in line with Gordy (2002) and Dietsch (2004). However, we claim that the simple comparison between different levels of asset correlations does not provide sufficient interpretative elements. Consider the case of Bankit data. The Regulatory asset correlation is equal to 13.9% while the higher estimated value is 3.7% (ML1). The difference is remarkable, but it is difficult to associate these numbers with any concrete interpretation, so it is not easy to understand which of the two is more realistic. The next Section aims at providing an interpretative scheme in which to insert these numbers.

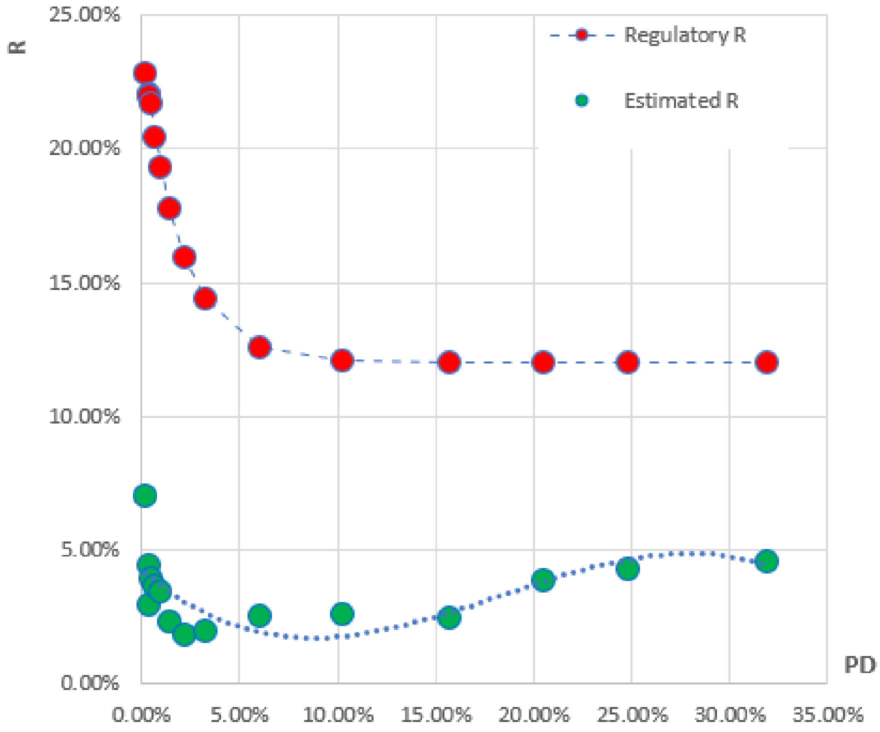
[11] The Cerved model has 10 rating grades. We aggregated some of them for this analysis.

Table 3: Regulatory and estimated asset correlations for each rating grade

Data Source	Rating	PD	Regulatory R	Estimated R						
				Method of Moments					Maximum Likelihood	
				MM1	MM2	MM3	MM4	MM5	ML1	ML2
Cerved	3	**0.05%**	23.68%	0.3%	0.7%	0.2%	0.7%	0.2%	0.9%	0.4%
	4	**0.18%**	22.97%	0.6%	0.8%	0.5%	0.8%	0.5%	0.7%	0.4%
	5	**0.52%**	21.25%	1.3%	1.4%	1.2%	1.3%	1.2%	1.2%	0.9%
	6	**1.36%**	18.07%	2.5%	2.6%	2.4%	2.3%	2.4%	2.4%	1.5%
	7	**3.19%**	14.43%	2.9%	2.9%	2.7%	2.6%	2.7%	2.6%	1.5%
	8	**6.14%**	12.56%	2.6%	2.7%	2.5%	2.2%	2.5%	2.6%	1.3%
	9	**12.79%**	12.02%	2.1%	2.1%	1.9%	1.7%	1.9%	2.0%	1.2%
	10	**42.23%**	12.00%	2.4%	2.5%	2.2%	2.0%	2.2%	2.5%	1.9%
Moody's	IG	**0.09%**	23.47%	11.1%	12.4%	10.7%	12.7%	10.5%	7.3%	7.2%
	SG	**4.26%**	13.42%	7.2%	7.4%	6.9%	7.3%	6.9%	7.5%	5.8%
Bankit		**3.70%**	13.89%	3.4%	3.4%	3.2%	3.1%	3.2%	3.70%	1.0%
Bank1	1	**0.06%**	23.64%	3.4%	12.2%	1.4%	11.0%	1.2%	1.4%	0.4%
	2	**0.17%**	23.02%	4.1%	6.9%	3.3%	6.3%	3.5%	5.0%	5.1%
	3	**0.34%**	22.14%	0.7%	2.3%	0.5%	2.1%	0.5%	3.6%	4.1%
	4	**0.69%**	20.48%	1.6%	2.4%	1.5%	2.3%	1.5%	2.8%	2.3%
	5	**1.30%**	18.26%	1.0%	1.5%	0.9%	1.4%	0.9%	1.8%	1.2%
	6	**2.54%**	15.37%	1.5%	1.9%	1.4%	1.6%	1.4%	2.0%	1.6%
	7	**5.09%**	12.94%	4.3%	4.6%	4.0%	3.5%	4.0%	5.0%	3.8%
	8	**10.85%**	12.05%	5.3%	5.4%	4.9%	4.2%	4.9%	5.7%	4.1%
	9	**30.13%**	12.00%	4.4%	4.6%	4.0%	3.8%	4.0%	4.6%	3.1%
Bank2	1	**0.21%**	22.83%	5.0%	6.6%	4.5%	6.3%	4.5%	7.0%	7.1%
	2	**0.35%**	22.06%	1.5%	2.3%	1.2%	1.8%	1.2%	3.0%	3.0%
	3	**0.37%**	21.99%	3.3%	3.9%	3.0%	3.7%	3.1%	4.5%	3.1%
	4	**0.43%**	21.69%	3.6%	3.8%	3.3%	3.6%	3.3%	4.0%	3.5%
	5	**0.70%**	20.47%	4.0%	4.2%	3.8%	4.0%	3.8%	3.7%	1.7%
	6	**0.99%**	19.33%	3.5%	3.6%	3.2%	3.5%	3.2%	3.5%	1.7%
	7	**1.46%**	17.78%	2.2%	2.3%	2.0%	2.1%	2.0%	2.3%	2.1%
	8	**2.21%**	15.98%	2.0%	2.1%	1.8%	2.0%	1.8%	1.9%	1.5%
	9	**3.23%**	14.39%	2.2%	2.3%	2.0%	2.2%	2.0%	2.0%	1.2%
	10	**6.01%**	12.60%	2.9%	3.0%	2.7%	2.9%	2.7%	2.6%	2.0%
	11	**10.20%**	12.07%	2.9%	2.9%	2.6%	3.0%	2.6%	2.6%	2.2%
	12	**15.69%**	12.00%	2.7%	2.8%	2.4%	2.8%	2.4%	2.5%	2.1%
	13	**20.45%**	12.00%	4.0%	4.1%	3.7%	3.6%	3.7%	3.9%	2.5%
	14	**24.84%**	12.00%	4.4%	4.5%	4.0%	3.6%	4.0%	4.3%	2.8%
	15	**31.90%**	12.00%	4.2%	4.6%	3.8%	3.4%	3.8%	4.6%	2.6%

Source: own elaboration

Figure 1: Regulatory vs Estimated (ML1) values of R, Bank 2



Source: own elaboration

4. The IRB and the stressed PD

Credit risk models: WCL and WCDR

In general terms, the risk measure, also known as Unexpected Loss (UL), is obtained from the difference between the Worst-Case Loss (WCL) and the expected loss (EL) that a portfolio can generate. The expected loss is obtained by multiplying the expected values of the probability of default (PD), the loss-given default (LGD) and the exposure at default (EAD[12]). The WCL represents the highest loss that is expected to be observed with a given level of confidence and its computation would require to know highest levels that are expected for the PD, LGD and EAD. A simplification is obtained by keeping constant the LGD and EAD and considering only the difference between the highest expected probability of default (the Worst-Case Default Rate, WCDR) and the average probability of default.

$$UL = WCL - EL =$$

[12] In general, the three terms PD, LGD and EAD could be not independent. In this case the average of the product does not necessarily coincide with the product of the averages. However, it is possible to assume that the three variables are conditionally independent. In other terms, their reciprocal dependency is shaped by a common factor and once this factor is fixed (realised) the variables are independent.

$$= WCDR * LGD * EAD - PD * LGD * EAD =$$

$$= [WCDR - PD] * LGD * EAD \tag{1}$$

Consider the simple introduction to credit risk modelling provided by the Chapter 15 - Section 8 of Hull (2015) where it is stated that under the gaussian Merton-Vasicek (MV) model, the Worst-Case Default Rate (WCDR), defined as the $(1-\alpha) - percentile$ of the distribution of the default rates, can be computed as:

$$WCDR = \Phi\left[\frac{\Phi^{-1}(PD)+\sqrt{R}\Phi^{-1}(\alpha)}{\sqrt{1-R}}\right] \tag{2}$$

where PD is the expected probability of default and R is the asset correlation. In practice, the WCDR is a given percentile of the default rate distribution: $\mathbb{P}(Dr > WCDR) = \alpha$. This expression is obtained under the hypothesis that the default of a borrower is triggered by another variable that can be either observable (the asset value of a firm) or latent (i.e. not observable variable and generically referred to as the creditworthiness).

The IRB model

The bulk of the BCBS proposal was the definition of the maximum or worst-case loss attainable given the level of confidence. If the LGD and EAD are defined as constants, the only variable is the probability of default (PD) and the WCL is equal to the product EAD*LGD*stressed PD where the stressed PD is provided by WCDR formula (1). Around this general framework, various proposals have been articulated in the context of the consultation process. In the 2001 proposal, in the risk weighting formula appeared the following expression (see BCBS (2001a)):

$$\Phi[1.118 * \Phi^{-1}(PD) + 1.288] \tag{3}$$

Notice that the (2) is a version of the (1) i.e. the MV formula for obtaining the stressed PD. Notice also that 1.118 = $1/\sqrt{1-0.2}$ and 1.288 = $\sqrt{0.2/(1-0.2)}\Phi^{-1}(0.995)$. This implies that the asset correlation was set at 20% and the confidence level at 99.5%.

Despite its centrality, the BCBS decided not to allow banks to use their own estimate of the asset correlation, mostly because the estimation of correlation parameters for defaults was, at the time, an area of evolving empirical and theoretical research, but also because the possibility to set the correlation values was seen as a policy instrument to be used for compensating for known limits of the models. It is also relevant to know that in this first proposal, the weighting formula was defined without subtracting the expected loss. In practice it was required to cover with capital the entire WCL.

Later and in response to feedback from practitioners, the Bcbs proposed an alternative formula for capital calculation, where the asset correlation was a decreasing function of the borrowers' probability of default and, for the firms, an increasing function of their dimension. For the calibration of the asset correlation for the Corporates portfolio, the BCBS applied two approaches – one direct and another survey-based or indirect.

Under the indirect approach, the BCBS collected from major banks around the world, information regarding their internal economic capital allocations against large corporate loans. For each institution, the data was used to estimate the implied risk weights (i.e. relative economic capital requirements) that each bank attributed to corporate loans having particular PD, LGD and maturity configurations. The BCBS also undertook a number of studies to independently estimate appropriate risk weights for large corporate loans using formal credit risk models (the direct approach)[13]. It is worth point out that the BCBS mentioned specifically that these studies (both the direct and the indirect approach) has been conducted only on data referred to large corporates.

[13] See the paragraphs 166-169 of BCBS 2001 b

The final proposed expression for the computation of the risk measure, also known as SF, can be expressed as follows[14] (for example as expressed in the Article 153 (1) of the CRR):

$$RW = \left\{LGD * \Phi\left(\frac{\Phi^{-1}(PD)}{\sqrt{1-R}} + \frac{\sqrt{R}\Phi^{-1}(0.999)}{\sqrt{1-R}}\right) - LGD * PD\right\} * \frac{1+(M-2.5)*b}{1-1.5*b} * 1.06 * 12.5 \qquad (4)$$

Having in mind the expression (1) it can be easily verified that the SF can be expressed as:

$$RW = \left\{\underbrace{LGD * WCDR}_{worst\ case\ loss} - \underbrace{LGD * PD}_{expected\ loss}\right\} * \psi \qquad (5)$$

Once α has been set to 0.999 and $\psi = \frac{1+(M-2.5)*b}{1-1.5*b} * 1.06 * 12.5$.

It can be also noticed that the LGD parameter is the same whether associated with either the average probability of default or the $(1-\alpha) - percentile$ of the distribution of the default rates. This observation is confirmed by Article 158 of the CRR which states that the calculation of the expected loss shall be based on the same input figures of PD, LGD and exposure value which are used for the computation of the risk weights. This implies that the expected loss is computed with the downturn LGD. This peculiarity is also highlighted in a note by BCBS (2005b) where it is said that in theory the expected loss should be associated with the expected LGD and not with the stressed LGD. It can be imagined that this approach depends on the difficulty encountered by banks, at least in the early years, in distinguishing the downturn LGD from the expected LGD. With this in mind, the expression (4), i.e. the SF, can be further simplified as follows:

$$RW = \{WCDR - PD\} * LGD * \psi \qquad (6)$$

This expression showcases the risk weight is essentially obtained as the difference between the WCDR, which is the $(1-\alpha) - percentile$ of the distribution of the default rates, and the expected value of the default rate scaled by the factor $LGD * \psi$.

The proposed weighting formula is based on models of the MV type with a single systemic factor. It was Gordy (2003) who clarified the conceptual basis behind this formula. One of the intentions was to make the capital requirement depend solely on the characteristics of the individual debtor whilst being independent from the portfolio composition. Gordy has shown that this result is obtainable only if the systemic factor is only one and if the portfolio is granular i.e. there is no concentration.

Since actual banking portfolios do not have the granularity characteristics required by the theoretical model, Gordy (2004) proposed a granularity adjustment as the final step in calculating the portfolio credit VaR. Granularity has to do with the degree of concentration of credit exposures: the finer the granularity of a portfolio, the greater the diversification and elimination of the idiosyncratic risk component of credit positions. Consistent with the Gordy scheme, the 2001 proposal of the Committee provides for an important adjustment for granularity (Granularity Adjustment) to take into account the fact that bank portfolios very rarely have the characteristics of perfect granularity.

With the proposal of April 2003 some changes are introduced with respect to the first version (BCBS, 2003a; BCBS, 2003b):

- The fixed correlation coefficient, R, of 20% is abandoned and a function, whose coefficient assumes a value between 12% and 24% depending on the value of the PD, is adopted. The function of the

[14] The maturity adjustment $[1 + (M - 2.5)]/[1 - 1.5 * b]$ reflects the assumption that mid-long term loans are more risky than short terms loans. b = (0.11852 – 0.05478*ln(PD))^2

correlation coefficient is modified in the case of small and medium-sized enterprises, to take into account the fact that SMEs are on average riskier than large companies.

- The increase in the confidence interval to 99.9% was chosen by the Committee to take into account the estimation errors that inevitably occur in the calculations of PD, LGD and EAD and model errors.

- The most important difference with respect to the previous proposal concerns the cancellation of the granularity adjustment, probably due to the considerable additional complexity that it would have imposed on banks, if adopted, in the context of an already very complex regulation. There had been numerous negative comments on this adjustment and several requests for simplification.

- The capital requirement is referred exclusively to the unexpected loss (UL), while it was recognized that the expected loss is absorbed by the provisions.

Obtaining the WCL from the risk weights is straightforward. The risk-weighted assets (RWAs) are obtained by multiplying the exposures at default by the risk weights. $RWA = EAD * RW$ where RW is the expression (4) or equivalently the expression (6). The amount 8%*RWA is the minimum required capital (MRC), and it coincides with the measure of the unexpected loss which is the difference between the WCL with a confidence level 99.9% and the expected loss. To obtain the WCL is then sufficient to sum to 8%*RWA the expected loss amount (ELA). This amount can also be named Total Loss[15], since it is the sum of unexpected and expected losses. This computation is not formally correct, because the risk weights formula also includes also the 1.06% factor and the maturity adjustment factor. However, 8%*RWA is actually the minimum required capital, while the expected loss must be covered by provisions or, in case the level of provisions is lower than the expected loss, by additional capital. Therefore, 8%*RWA+ELA is in practice the amount of capital and provisions that the Regulation requires to hold.

Consider the following example where an exposure of 1 euro million is associated with PD equal to 1% and LGD equal to 25%; the expected loss then amounts to simply 1%*25% = 0.25% multiplied by the EAD, which implies that, in normal conditions, it can be expected to loss 2,5 thousand of euro (0.25%*1 euro million). The WCDR with a confidence level 99.9% is equal to 14%; this implies that the expected probability that the default rate will exceed this level is 0.01% and that the WCL is computed by assuming that the default rate is equal to this value. The unexpected loss (8%RWA) is equal to 34,5 euro thousand and it represents the maximum deviation from the expected loss that is expected. Finally, the WCL in monetary terms is equal to 37 euro thousand (3.7% of the EAD) obtained by summing the expected and unexpected loss.

It should be noticed that what does practically mean that the risk weight is 43.2% is not so immediate, while knowing that the WCL is 3.7% (or 37 € thousand) gives a more natural point of view. Further, by supposing that this exposure is backed by 50 € thousand of capital, the capital on RWA ratio would be 11.6%, which is higher than the regulatory minimum (8%) but it should be recognized that it is not so easy to understand what it does represent in concrete that the difference between the ratio and the minimum is 3.6%? Instead by comparing the capital with the WCL, it is revealed that the capital amount would be sufficient to cover the estimated WCL (generated by the occurrence of a default rate of 14%) plus additional 13 € thousand of losses.

[15] See also Cannata (2020)

Table 4: Example of computation of the WCL from regulatory figures

EAD	**€ 1,000,000.0**
LGD	**25.00%**
M	**1**
PD	**1.00%**
R = Eq. (7)	19.28%
WCDR = Eq. (2)	14.03%
maturity adjustment	1
ELA = EAD*PD*LGD	€ 2,500.0
RW = Eq. (4)	43.20%
RWA = EAD*RW	€ 431,528.2
MRC = 8%RWA	€ 34,522.3
WCL = MRC + ELA	€ 37,022.3
WCL/EAD	3.70%

The asset correlation and the WCDR (or stressed PD)

For illustrative purposes, in this section we show in practice how it is possible to obtain the stressed PD i.e. the estimate of the WCDR using the MV model. This is done both by estimating the asset correlation from real data, then exploiting the asset correlation provided by the BCBS for the SF. This enables to verify how the value of the asset correlation influences the estimated WCDR.

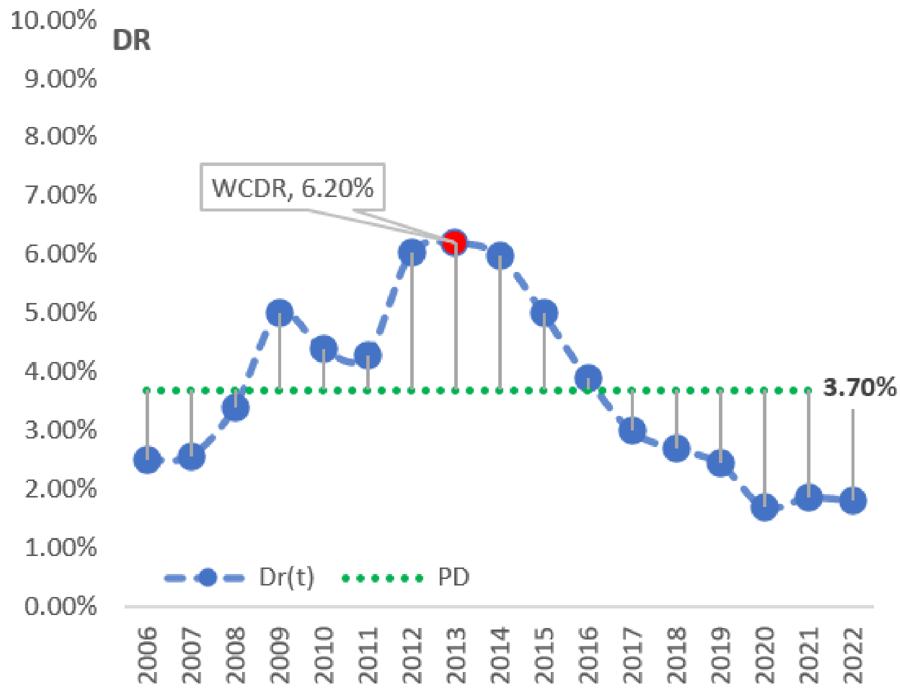
In this section we use the publicly available data provided by Bankit. It is worth mentioning that we are working with aggregate data while the regulatory formula applies at a more granular level. However, here we are focusing the attention on the computation of the estimated WCDR only. In Section 5, where we extend the analysis to the computation of the estimated portfolio WCL, we work with counterparty level data.

Observed WCDR

The following chart represents the time series of the annual default rates of non-financial firms in Italy. The average of these 17 points is 3.70% and this represents the estimated average probability of default (long run PD). The observed WCDR, as of 2013, is equal to 6.20%. We could use this number as the estimate of the stressed PD thus the measure of risk i.e. the unexpected loss would be proportional to the quantity 6.20% - 3.70% = 2.51% (i.e. observed WCDR minus the average of the default rates).

Say for example that the LGD is equal to 50%; leaving aside the factor ψ for simplicity, we have (see Eq. (5)): Worst Case Loss = 6.20%*50% = 3.10% and Expected Loss = 3.70%*50% = 1.85%. The unexpected loss would be (6.20% - 3.70%)*LGD = 2.51%*50% = 1.25%. Said in other terms, for a portfolio of 1 mln of euro, the Worst-Case Loss would be 31,022 € of which 18,494 € is the expected loss component, while the unexpected loss (that should be covered by the capital) would amount to 12,528 €.

Figure 2: annual default rates for non-financial firms in Italy



Source: Banca d'Italia

The observed WCDR could be considered as not sufficiently prudent. Even if the observation period includes the worst financial crisis of the last few decades, it is possible that the time series was too short to give the possibility to observe the 99.9-esim percentile of the distribution of the default rates. For this reason, we need a model.

Estimated WCDR

Both the reduced form and structural models provide the entire probability distribution of the default rates from which we can derive any desired quantile. Both approaches rely on two parameters i.e. the expected value of the default rates $\mathbb{E}(Dr)$, usually named unconditional or long run PD and estimated as the simple average of the default rates, and the default correlation (for the reduced form models) or the asset correlation (for the structural models). Under the MV model, the expression (2) provides the α percentile given the parameters PD and R. PD is estimated as the average of the observed default rates which we know (see Figure 2) being equal to 3.70%.

With the data underlying Figure 2 and using the Maximum Likelihood estimator proposed by Duellman (2004), it is possible to obtain an estimate of R equal to 0.03697 (see Table 3, ML1). Setting $\alpha = 99.9\%$ the expression (2) produces 11.21%, this being the estimate of the WCDR. In other terms, we expect that the probability of the default rates exceeding the value of 11.21% is only 0.1%. As it can be appreciated from Figure 3 - a, the estimated WCDR is well higher than the observed WCDR. With an LGD equal to 50%, the Worst-Case Loss is 11.21%*50% = 5.61% which is 56,068 € with a portfolio of 1 mln euros. The unexpected loss (and then the capital requirements) is (11.21% - 3.70%)*50% = 3.76% i.e. 37,573 €. This amount must be compared with 12,528 €, which was the unexpected loss obtained referring to the observed WCDR.

Regulatory WCDR

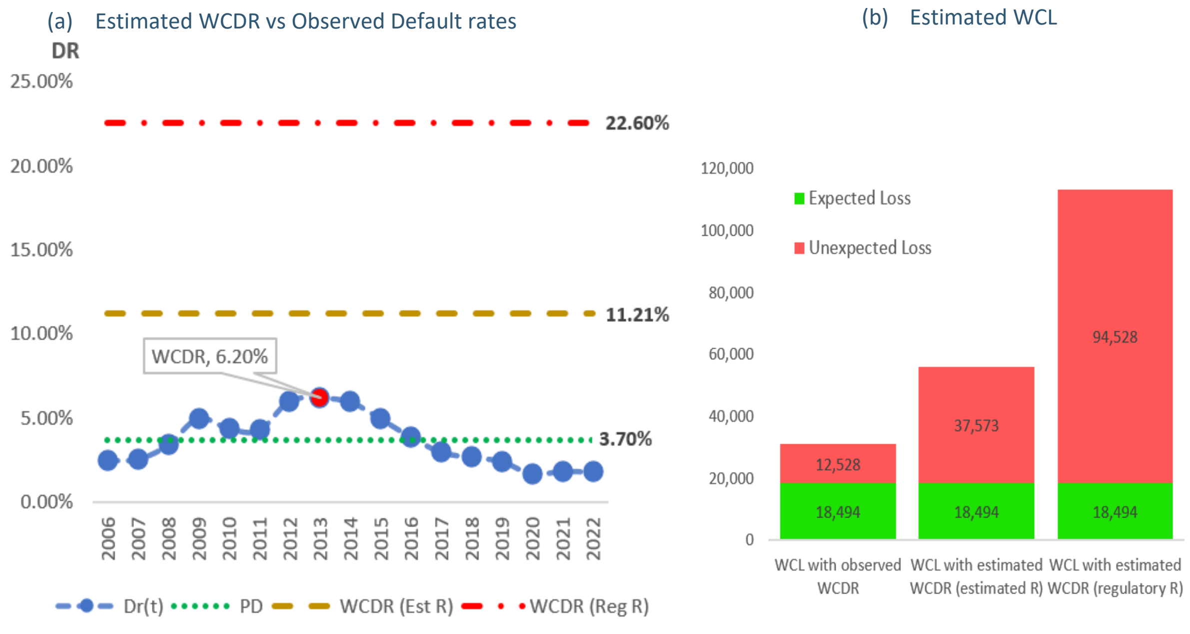
Now we use the regulatory[16] value of R. Under the IRB framework, for the Corporates portfolio, the asset correlation is a function of the PD:

$$R = 0.12 * \frac{1-e^{-50*PD}}{1-e^{-50}} + 0.24 * \left(1 - \frac{1-e^{-50*PD}}{1-e^{-50}}\right) \tag{7}$$

With PD = 3.70% the expression (7) provides for R a value equal to 0.1389. This is the regulatory estimate of the asset correlation. Feeding the expression (2) with this parameter, the estimated WCDR is 22.60%. With an LGD equal to 50%, the worst-case loss for a portfolio of 1 mln euros is now 113,022 € and the unexpected loss is 94,528 €.

The following figure provides the comparison between the observed default rates and the estimated WCDR, obtained with the same model but with different values of the R parameter. It is also shown the difference in terms of risk measures expressed in monetary terms. It is important to understand that with fixed LGD and exposures (1 mln in this case) there is a strict connection between the risk measure (from which the capital requirements are derived) and the level of the stressed PD (or estimated WCDR) provided by the model.

Figure 3: Observed vs estimated WCDR for non-financial firms in Italy and implied loss for a 1 mln portfolio



Source, own elaboration

The following table synthetises the results obtained. The observed WCDR is the highest default rate observed in the time series used for the estimation of the PD. For example, for the time series of the default rates of the Moody's class Investment Grades (IG), the observed WCDR in 42 years was 0.62% against an average equal about 0.09%. The estimated WCDR is obtained with the expression (2) that is the core of IRB SF using the regulatory asset correlation (i.e. expression (7)) or using the estimated asset correlation. Considering again the Moody's class IG, it can be seen that the estimated WCDR obtained with the regulatory asset correlation is

[16] Cfr Article 153 (1) of the CRR

2.49% (4 times the observed WCDR) while the estimated WCDR obtained with the estimated asset correlation is 1.54% (2.5 times the observed WCDR).

Table 5: WCDR Observed vs Estimated

Data Source	Rating	PD	WCDR		
			Observed	Estimated	
				Regulatory R	Estimated R
Cerved	3	**0.05%**	0.10%	1.28%	0.14%
	4	**0.18%**	0.30%	3.35%	0.41%
	5	**0.52%**	0.94%	7.31%	1.34%
	6	**1.36%**	2.64%	13.38%	4.14%
	7	**3.19%**	6.01%	20.84%	8.97%
	8	**6.14%**	10.21%	29.46%	14.67%
	9	**12.79%**	19.71%	44.89%	24.44%
	10	**42.23%**	51.84%	80.93%	61.70%
Moody's	IG	**0.09%**	0.62%	2.49%	1.54%
	SG	**4.26%**	12.09%	25.06%	18.10%
Bankit		**3.70%**	6.20%	22.60%	11.21%
Bank1	1	**0.06%**	0.34%	1.79%	1.07%
	2	**0.17%**	0.55%	3.57%	1.40%
	3	**0.34%**	0.70%	5.64%	1.66%
	4	**0.69%**	1.39%	9.00%	2.42%
	5	**1.30%**	1.92%	12.93%	3.38%
	6	**2.54%**	3.91%	18.43%	6.28%
	7	**5.09%**	9.42%	26.99%	16.55%
	8	**10.85%**	17.95%	41.62%	30.43%
	9	**30.13%**	40.96%	70.57%	55.78%
Bank2	1	**0.21%**	0.66%	4.14%	1.69%
	2	**0.35%**	0.61%	5.74%	1.43%
	3	**0.37%**	0.81%	6.01%	1.89%
	4	**0.43%**	1.02%	6.64%	1.99%
	5	**0.70%**	1.54%	9.19%	3.12%
	6	**0.99%**	2.03%	11.28%	3.79%
	7	**1.46%**	2.65%	13.87%	4.20%
	8	**2.21%**	3.83%	17.18%	5.70%
	9	**3.23%**	5.49%	20.85%	8.10%
	10	**6.01%**	10.79%	29.15%	15.00%
	11	**10.20%**	17.13%	39.57%	22.65%
	12	**15.69%**	24.97%	50.49%	30.91%
	13	**20.45%**	32.30%	58.44%	42.03%
	14	**24.84%**	36.41%	64.47%	49.11%
	15	**31.90%**	42.97%	72.39%	57.78%

Source, own estimation

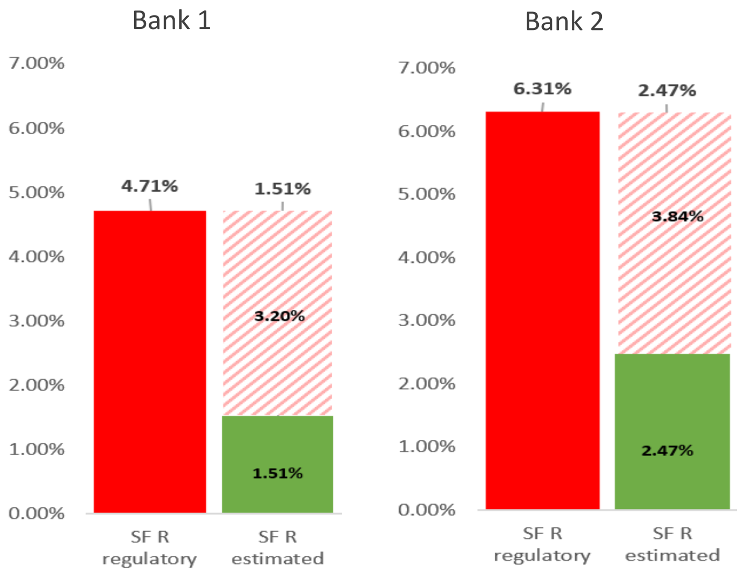
5. Estimated vs regulatory WCL

Estimated vs Regulatory WCL

In this section we compare the WCL computed as total loss (expected and unexpected loss) obtained with the SF and the calibration of the asset correlation provided by the equation (7) i.e. the regulatory calibration of the SF, with the total loss obtained with the same SF but with the estimated value of the asset correlation. Using the borrower level data stemming from the two banks' Corporate portfolios, we computed the RWA at single name level with the expression (4) and obtained the Total Loss contribution of each borrower as 8%*RWA + Expected Loss Amount, then we summed the results. All the borrowers classified in a given rating grade shared the same probability of default and consequently the same value of R was assigned both when using the regulatory and the estimated values for the asset correlation. The LGD and exposures at default were instead different and specific for each borrower.

We obtained the sum of the Total Loss computed at borrower level, then we scaled this amount by the sum of the exposures at default. The results are represented in the next figure for the two banks. The Total Loss (i.e. the estimated WCL observable with 99.9% of confidence level) is 4.71% and 6.31% of the exposure value. When the regulatory value of the parameter R is substituted with the estimated value, the Total Loss decreases to 1.54% and 2.47% of the exposure value. In relative terms, the regulatory figures (obtained with the value of R provided by the expression (7)) are about three times (3.1 and 2.6 respectively) the values obtained with the same model (the SF) using instead the estimated asset correlation[17].

Figure 4: Total Loss for the Corporates portfolio with the SF: regulatory vs estimated asset correlation



Source: own estimation on banks internal data

The difference we obtained in terms of Total Loss is considerable and deserves some additional thought. First, we have seen in section 2 that derivation of the SF relays on the hypothesis that the portfolio is not concentrated

[17] Also in Dietsch (2013) it can be found, with reference to the French banking system, that the regulatory minimum capital requirements are about two times the level of the capital that can be obtained by fully calibrating a portfolio credit risk model.

(perfectly granular). This hypothesis is quite unrealistic given that data refer to a Corporates portfolio. It is thanks to the infinite granularity hypothesis that the measure of risk can be obtained with a closed formula like the SF.

The concentration effect

Abandoning the hypothesis of perfect granularity implies either correcting the SF with an adjustment like the one proposed by Gordy or switching to simulation-based methods. In order to verify the impact of the concentration on the risk measures, we used a Monte Carlo (MC) approach. In practice, the results obtained with the MC approach should approximately be equal to the results obtained with the SF in case the concentration of the portfolios (the single rating grades) was low. In case the concentration is high, the MC approach should return a higher measure of risk.

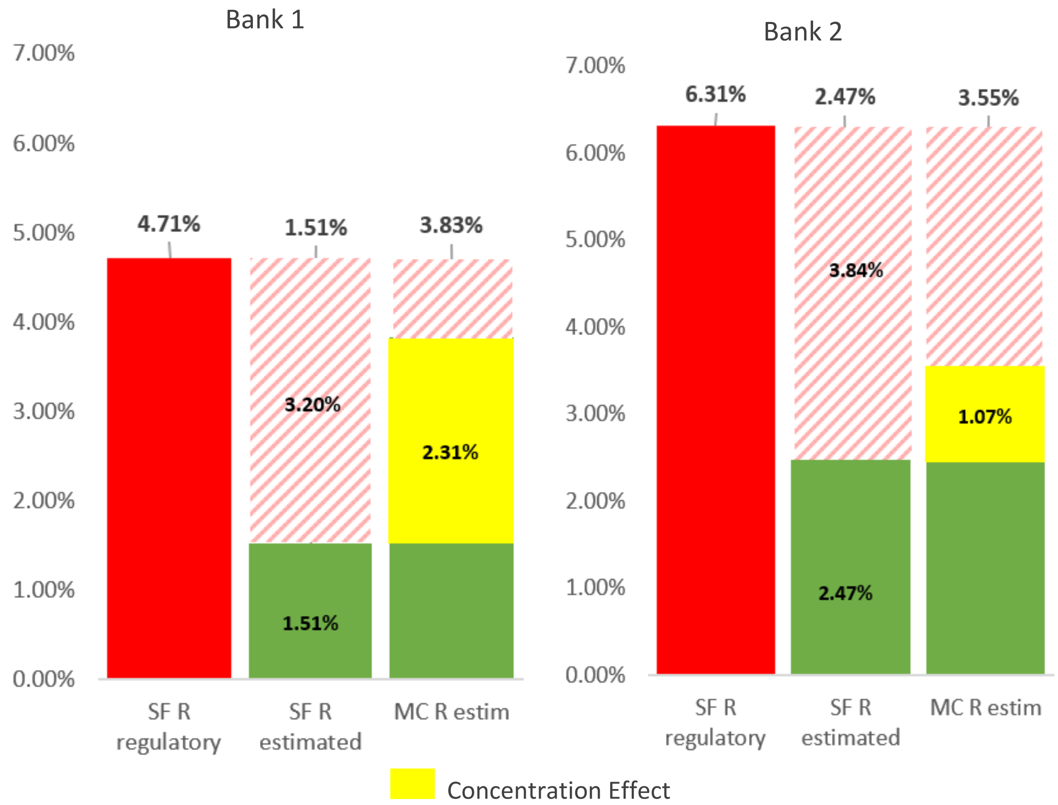
Here we briefly depict how the MC approach works. The SAS codes that have been used are included in the Annex. The values y and z of the common factor Y_t and of the idiosyncratic factor $Z_{i,t}$ are generated from independent standard normal variables. The creditworthiness of each borrower of the DS2 is computed with the parameter R estimated for each rating class (see Table 3). Each borrower is then considered as defaulted if his credit worthiness is less than $\Phi^{-1}(PD)$ with the PD depending on the rating class. Summing the loss (exposure*LGD) of all the defaulted borrowers produces the total losses obtained given the scenario y. This process is repeated many times. In this way it is possible to obtain the entire probability distribution of the total losses, thus letting us derive empirically the $\alpha - esim$ percentile of this distribution. We repeated this procedure for each rating grade and then summed the result for each bank[18]. To stabilise the results, we exploit a variance reduction technique consisting in sampling for Y_t more heavily from the left tail of the normal distribution and then reweighting the results to obtain correct measures[19]. The results we obtained were indeed quite stable.

The impact of the concentration effect appears wide for both banks (see the figure below). In detail, the regulatory Total Loss is now only 1.2 and 1.8 times (respectively for the two banks) higher than the measure obtained exploiting the MC approach with the estimated value of R. These results can lead to think that the regulatory measures (SF with regulatory R) already include a correction for the concentration effect and this correction is embedded in the quantification of the asset correlation.

[18] Summing the α −VaR obtained for each rating class it is equivalent to assume that the risks in the rating grades are perfectly correlated that is, there are not beneficial effects due to portfolio diversification. This is in line with the general setting of the Basel II that for example does not recognize the possible diversification effect due to segmentation of the portfolio by countries or sectors of the counterparties. This setting, in turn coherent with the single risk factor hypothesis, allows to obtain a considerable simplification. The scenarios (the y values) where re-generated for each rating class. We also tried to keep the same scenarios for all rating classes and the results were substantially the same.

[19] See Bolder (2018) chapter 8

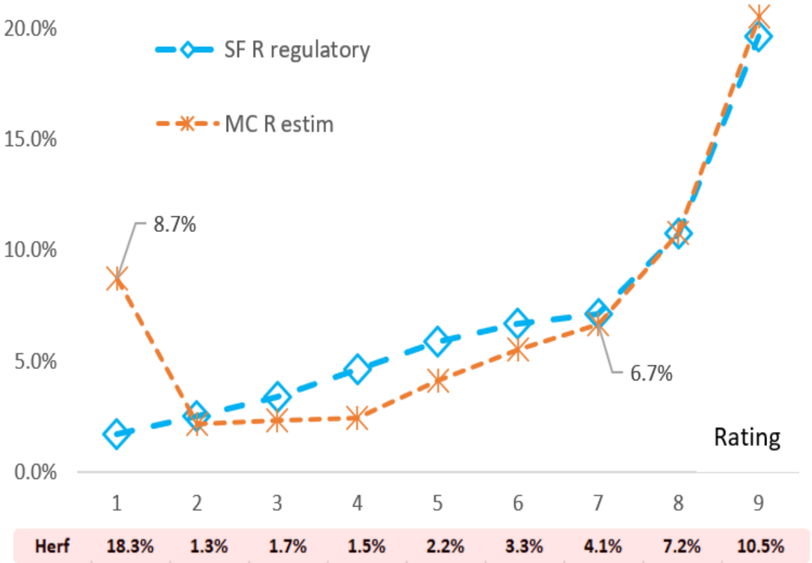
Figure 5: Total Loss obtained with the SF (regulatory and estimated asset correlation) and WCL obtained with Monte Carlo simulations and estimated asset correlation



A better understanding is provided by comparing the two measures of risk at rating grade level. The next figure shows the level of the Total Loss obtained with the SF using the regulatory calibration of R for each rating grade of the Bank1 portfolio and the WCL obtained with the MC approach but with the estimated R. It is also shown the Herfindahl index (a measure of the concentration) computed at rating level. It can be seen that for most of the rating grades the MC measure of risk is lower than the SF measure. This implies that the concentration effect is not sufficient to compensate for the reduction of R (see Table 3). It can also be noticed that the Herfindahl index for these rating grades is quite low. However, for the first and the last rating grade the MC measure is higher than the SF measure and this occurs in correspondence with higher values of the Herfindahl index. What happens in these cases is that the concentration effect is able to more than compensate for the reduction of the asset correlation.

It is also notable to see how the risk measures may change dramatically when the concentration effect is considered. The rating grade 7 has PD which is about 50 times higher than the PD of the rating grade 1. However, the measure of risk obtained with the MC approach (i.e. considering the concentration effect) is higher for the rating grade 1 (8.7% vs 6.7%). Moreover, for the first and last rating grade, where the concentration appears high (Herfindahl > 10%), the risk measure obtained with the MC approach and the estimated asset correlation is higher than the regulatory measure. This implies that in some cases, the regulatory calibration of the asset correlation may not adequately compensate for the lack of consideration of the concentration effect.

Figure 6: Total Loss with the SF (regulatory asset correlation) vs WCL with the MC simulations (estimated asset correlation) at rating grade level, Bank 1



Other corrections

We now briefly consider other shortcomings of the MV model and the related corrective measures. It is known that the estimation of the asset correlation encounters some limitations (see for example Gordy (2002)). One of these limitations is represented by the hypothesis that all the borrowers classified in a given cluster or rating class have the same level of probability of default. If this hypothesis is not satisfied, i.e. if the borrowers have different probability of defaults, then the resulting estimated asset correlation will be biased. In particular, it is known that the estimates are downward biased. Resti (2008) suggested to multiply the estimated asset correlation by 1.5. Based on this analysis, this factor should ensure robust estimates even in the case the hypothesis of homogeneous probability of default is violated. The other limitation is the length of the time series of default rates. With short ranges of years of observation (less than 20, according to Gordy) of the default rates, the estimates are known to be downward biased.

In our case, we are working with borrowers classified in rating grades through a scoring system that includes several risk indicators and expert evaluations. We can then expect that the borrowers in each rating grade are quite homogenous in terms of probability of default, but still some sources of heterogeneity may exist. More serious is the problem of the length of the observation period. The number of years of observation of the default rates at our disposal is only 14. For these reasons we decided to apply a corrective factor as the one suggested by Resti and, namely, we adopted a factor equal to 2[20]. Notice for example that with this correction, the asset correlation for the first rating grade becomes 11.68%*2 = 23.36%, which is not so far from the regulatory value equal to 23.41% (see Table 3). However, for the other rating grades, the difference is still wide.

Another subtle shortcoming of the gaussian MV model relates to a quite technical aspect known as tail correlation. The main feature of the MV model is to provide a method to generate a predefined level of correlation among the default events. However, it can be shown that as we increase the confidence level (α), the correlation between default events tends to decrease and for high α levels this reduction is important. This problem is strictly related to the hypothesis that the common factor Z_t has normal distribution. An approach to overcome this problem consists in abandoning the normal distribution and adopt another one, such as the T distribution.

[20] A similar expedient has been used also in Tola (2010)

The departure from the Gaussian hypothesis for the common factor deserves some attention[21]. The MV is based on the normal distribution because the log-yields of the activities are by assumption described with the geometric Brownian motion (which in essence is a transformation of the normal standard); if we abandon the normal distribution, to have standardized returns linked by a t-copula (and not by a Gaussian copula that has zero correlations in the upper and lower tails), for consistency's sake, it is also necessary that the calculation of the distance-to-default is adjusted and the same is true for the estimate of the conditional PD which must be obtained referring to a T-distribution. It follows that the regulatory formula should be updated to insert the inverse of the t-distribution for the PD and the common factor. For example, the 99.9% quantile of a standard normal variable is -3.09 while the same quantile of the Student t with 3 of freedom is about -10.21.

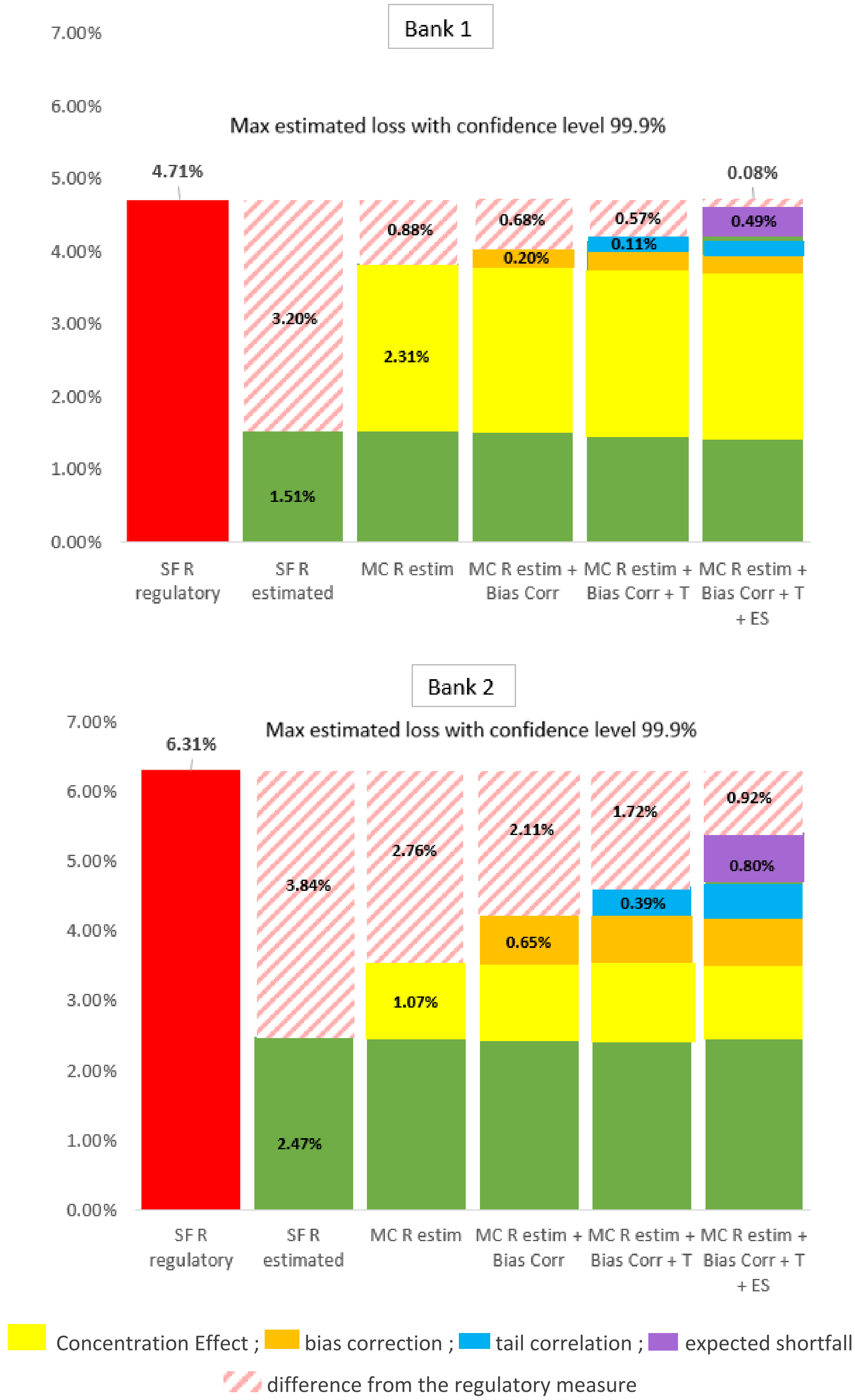
With the aim of controlling also for this problem, in our MC exercise, we multiplied the firms' asset value by the random variable $\sqrt{v/W_{i,t}}$ where $W_{i,t}$ is a $\chi^2(v)$ random variable with v degrees of generated freedom. In practice, $\sqrt{v/W_{i,t}}\mathrm{r}_{\mathrm{i,t}}$ is a standard T-distributed random variable with v degrees of freedom. The degrees of freedom have been set so as to reproduce a default correlation equal to two times the estimates obtained with one of estimators in Table 3 and a level of tail dependency equal to 5%. With the aim to better exploit the fat-tail characteristic of the T distribution, we also computed the Expected Shortfall (ES) or tail VaR as the average of the losses exceeding the VaR. This further step can be considered as a heuristic way to account for other not-easily quantifiable shortcomings of the underlying model as the model risk.

Lastly, besides the volatilities and the correlations, an additional feature of default rates that is commonly empirically observed but not accounted for by the class of models backing the SF is the persistence of the shocks. The persistence, measured by the serial correlation or autocorrelation, is the ability of shocks to reflect their effects even in periods subsequent to their occurrence. Adopting a one-year perspective brings to implicitly assume that the effects of a shock (adverse scenario) observed in a given period, do not spread over the next periods. Default data referred to SMEs and Households could reveal a lower-than-expected volatility but a quite high persistence that the SF is not suited to account for. We have not implemented this correction because a different class of models, which makes it possible to take into account the dynamics of default rates, should be considered.

Figure 7 shows the results for the two banks. Now it is possible to retrieve the level of the TL obtained with SF and the regulatory value for the asset correlation. The correction to the estimated asset correlation obtained by multiplying the estimates with a factor of 2, produces an increase of the risk measure by some percentage points. The impact of substituting the normal distribution with the T distribution for the generation of the common factor has a more material impact.

[21] See Bolder (2018) section 4.3, 4.4

Figure 7: Total Loss with the SF (regulatory and estimated asset correlation) and WCL and ES with Monte Carlo simulations with estimated asset correlation multiplied by 2 and imposing a 5% of tail dependency



6. Conclusions

The level of capital requirement generated by the IRB approach depends crucially on the asset correlation – a parameter that enters the regulatory risk weight formula and is determined by the Regulators. Several studies have estimated the asset correlation and found values lower than the regulatory calibration. Exploiting different data sources, we confirm the findings from existing literature. However, we believe that the simple comparison between the estimated and Regulatory values of this parameter does not offer an easy interpretation. We then propose a different point of view.

The IRB Regulatory Supervisory Formula (SF) can be seen as an algorithm taking the long-run PD as input and providing the stressed PD as output. Looking at the stressed PD provides a natural interpretation of the risk measure produced by the SF. In fact, it is possible to compare the obtained stressed PD with the historical series of the default rates. From this comparison, it is possible to appreciate the level of prudence inherent in the SF. It is also possible to translate the Risk Weighted Assets into estimated Worst-Case Loss (the highest economic loss that can be expected with a given level of confidence) and this provides an easier economic interpretation of the obtained risk measures.

We compare the regulatory risk measure with the risk measures obtained by substituting, in the SF, the regulatory asset correlation with the one that can be estimated from observed default rates. At least for the data at our disposal, justifying the observed difference in the risk measure, requires retrieving the concentration effect plus other margins of prudence. In other terms, we have verified that the current regulatory calibration of the SF seems to already account for a series of known limits of the reference model that the Basel Committee avoided formalizing for sake of simplicity such as the hypothesis of perfect granularity and the uncertainty related to the structure of the default correlation. This implies that while the asset correlation input seems overestimated and the effect of concentration seems not considered, the aggregate risk measure does not seem mis-calibrated if both these inputs are correctly accounted for.

While the empirical results presented cannot easily be extended outside the analyzed sample that it is limited to two banks, they demonstrate the advantages of the proposed method that, by focusing the attention on the stressed PD and Worst-Case Loss instead of on unexpected loss multiplied by 12.5 (i.e. the Risk Weighted Assets), enables to disentangle the risk components (like the concentration effect). This kind of analysis could be included in the Pillars 3 of the IRB validated banks for transparency, benchmarking and better comprehension of these metrics by the market participants.

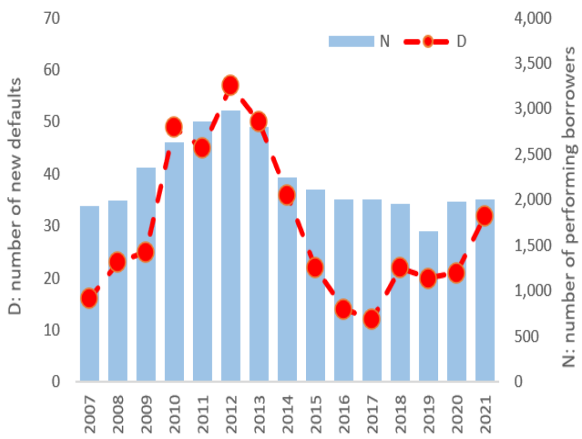
7. Annex

The variance of the default rates: a concrete example

Consider the following simple example built with real data22 coming from the Corporates portfolio of an Italian bank. The chart below represents the number of borrowers (performing at the beginning of each year - N_t) and the number of new defaults (D_t) observed during the year t for a given rating class or grade of the portfolio. Thanks to this data, it is possible to compute 15 annual default rates ($Dr_t = D_t/N_t\,; t = 1,2 \dots 15$), and the (simple23) average of these default rates is an estimate of the long run probability of default for this rating grade. The average of the default rates is equal to 1.273%. The number of borrowers is quite stable across the years and the average number of borrowers is equal to 2235. We then have an estimate of the parameters p and N of a binomial distribution and we can derive the expected variance as 1.273%(1-1.273%)/2235 = 0.00056%.

We can also compute the variance empirically and the result is 0.00166%. The empirically observed variance is 3 times the variance of a binomial random variable with parameters $N = 2235$ and $p = 1.273\%$ and this result (i.e. the observed variance of the default rates being higher than the variance implied by the hypothesis of independence) is commonly found when dealing with credit portfolios. This evidence, in turn, casts doubts on the hypothesis that the defaults are independent and consequently on the opportunity to employ the binomial model in this context.

Figure 8: New defaults and Number of borrowers for a rating grade of a Corporates portfolio



Source: bank internal data

$$Average\ N = 2235$$

$$Average\ Default\ rate = \frac{\sum_{t=2007}^{2020} Dr_t}{15} = 1.273\%$$

$$Variance\ of\ a\ Bi(N = 2235, p = 1.273\%) = 0.00056\%$$

$$Estimated\ Variance = \frac{\sum_{t=2007}^{2020}(Dr_t - 1.273\%)^2}{15} = 0.00166\%$$

[22] The details about the data used are in the Section 4.

[23] Article 180 (1) (a) of the Regulation (EU) No 575/2013 (CRR) states that << *institutions shall estimate PDs by obligor grade from long-run averages of one-year default rates* >> but it is not clear whether the average should be simple or weighted. However, in the EBA (2017) guidelines paragraph 81 it specified that institutions should calculate the observed average default rates as the arithmetic (i.e. simple) average of all one year default rates.

Monte Carlo simulation of the MV model

What follows is an example of Sas code that can be used to obtain a Monte Carlo evaluation of the VaR at given confidence level under the MV model. The code exploits a variance-reduction technique, in practice the scenarios Y_t are extracted from a normal distribution shifted toward negative numbers.

The data set <<rating>> contains the data at borrower level referred to as a given rating grade (the PD is the same for all the borrowers). In particular, the variable *severity* is built as EAD*LGD. The program generates a 1xM vector of scenarios (extractions from a normal distribution with a central parameter equal to &med) and the NxM matrix of the idiosyncratic effects (extracted from a standard normal distribution) where N is number of clients in the rating grade. Z_E is the matrix containing the creditworthiness computed for each borrower and under each scenario. D is a (0,1) NxM matrix where 1 is assigned when the creditworthiness is less than $\Phi^{-1}(PD)$. LZ is a Mx2 matrix in which the first column contains the loss rate observed under each scenario and the second column contains the realized scenarios.

In the data step, after the iml procedure, it is computed the weighting factor (wgt) of each scenario. This passage is needed to correct for the fact the scenarios were not extracted from a standard normal distribution. The univariate procedure is then used to compute the percentile of the distribution at the desired level of confidence.

```
%let med = -1.4;
data _null_;
    set rating end=eof;
       if eof then threshold= probit(PD);
       if eof then call symput ('Thr',threshold);
end;
proc iml;
    use rating;
       read all var {severity} into A[colname = varNames];
       call randseed(-1);
       Z = j(1,&M);
       E = j(&N,&M);
       call randgen(Z,"Normal",&med,1);
       call randgen(E,"Normal",0,1);
       Z_E = sqrt(&ro)*Z+sqrt(1-&ro)*E;
       D = (Z_E<&Thr);
       LZ = (A`*D)`/&S_ead ||Z`;
       create Ldistr from LZ[colname={"L" "Z"}];
              append from LZ;
       close Ldistr;
quit;

data Ldistr(drop=Z f g);
       set accum end=eof;
       retain S_wgt 0;
       f=PDF('NORMAL',Z,0,1);
       g=PDF('NORMAL',Z,&med.,1);
       wgt = f/g;
run;

proc univariate data=Ldistr noprint;
       var L;
       weight wgt;
       output out=Pctl_TL(keep= P_95 p_98 P_99 P_99_5 P_99_7 P_99_8 P_99_9)
                      pctlpre=P_ pctlpts= 95 to 100 by 0.1;
run;
```

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ACKNOWLEDGEMENTS

We are grateful to V. Bonten, F. Cannata, G. Cascarino, S. Cuneo, L. Giaccheirni, A. Ilari, R. Kopp, A. Leardi, D. Mokas, M. Quagliariello, M. Reggi, A. Resti, G. Rinna, R. Russo and E. Sabatini for useful comments and suggestions on a preliminary draft of the paper

Simone Casellina:
Bank sector analyst, European Banking Authority

Fabio Salis:
Chief Risk Officer, Creval

Giovanni Tessiore:
Head of Risk Advanced Analytics & Innovation, Cerved

Roberto Ugoccioni:
Senior Model Expert, Intesa San Paolo

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ISBN 978-92-9245-923-9
ISSN 2599-7831
DOI 10.2853/080166
DZ-AH-23-003-EN-N