Bank risks and liquidity dynamics: evidence from the euro area financial crisis

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All opinions expressed in the presentation are mine. They do **Not** reflect the Bank of Italy’s views.
BACKGROUND FACTS

• The financial crisis heavily affected the euro area interbank money market, by increasing liquidity risk and counterparty credit risk: according to the ECB's Euro Money Market Survey (2010), euro money market aggregate turnover decreased, as a consequence of interbank trades contraction

• The ECB introduced extraordinary measures: fixed-rate tender procedure with full allotment in ordinary three-month LTROs; two 3-year, 12-, 13- and 6-month full-allotment LTROs; full-allotment fixed-rate MROs; reduced the reserve ratio from 2 to 1 percent; broadened the set of collateral eligible for refinancing operations

• ECB's Financial Stability Review (2015, 2017) argues that banks may have replaced more expensive debt funding with Eurosystem's financial support

• Banks’ price-to-book ratios have decreased to low levels, reflecting market doubts about banks' asset profitability, mostly stemming from long-standing non-performing loans
OVERVIEW: OBJECTIVE

The study suggests a methodology, using data from the financial crisis, to curb banks' daily liquidity risks:

• Liquidity risks are modelled as euro area banks and ECB's reactions to innovations, in the form of unexpected changes of traded liquidity amounts
• The equilibrium condition between liquidity demand and supply dynamics is empirically tested, to estimate euro area bank risks' evolution and persistence
• Modelling reactions to innovations and estimating liquidity shocks' size help understand how big liquidity buffers should be in a real-like stress scenario
• Estimated liquidity shocks and sensitivities can be used for stress-testing purposes in order to assess liquidity buffers' adequacy
OVERVIEW: MAIN FINDINGS

• Euro area banks' liquidity dynamics, sustained by ECB's accommodative supply, has been particularly sensitive to stress-induced shocks, exhibiting very long memory of lagged conditional variances and disturbances.

• Expectations of risk-related variables appear to explain a large fraction of banks’ liquidity dynamics.

• Estimation results show that banks' precautionary liquidity buffers, at the 0.95 confidence level, should be added 11.3 percent to the daily expected liquidity change; they should be increased by 21.9 and 44 percent for reaching 0.975 and 0.99 confidence level.
Bank’s Problem: Intro

Under uncertainty, the bank retains cash to settle daily payment obligations as long as excess liquidity does not harm the bank’s own profitability, for missing yielding investments.

Specifically, the bank’s problem is to select the amount of liquidity over time so as to minimise the cost of payment failures, i.e. the liquidity risk, against the opportunity cost of holding cash.

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The bank chooses cash $L_s \geq 0$ to minimise $c(L_s)$ over discrete time $s$, i.e.

$$V(L) = \min_{\{L_s, L_{s+1} | s \in F(J_s)\}} \left\{ \infty \right\} c(L_s) + \beta^{s+1} E_s V(L_{s+1})$$  \hspace{1cm} (1)

s.t. $L_{s+1} = L_{s+1} | s + \zeta \eta_{s+1}, \zeta > 0, \eta_{s+1} \sim (0, \phi_{s+1})$

where $c(L_s) = (A_s R_s - Y_s L_s)^2$, $A_s$ is the amount of assets bearing financial risks $R_s$, $Y_s$ is the foregone yield for holding cash $L_s$, $0 < \beta < 1$ is the discount factor and $E_s$ the expectation operator, given information available at time $s$.
Bank’s Problem: Rationale

The bank holds cash just enough to offset the cost associated with the liquidity risk and prevent profitability from suffering excess liquidity. Hence, the bank wants the illiquidity cost and the opportunity cost to be as close as possible, i.e. the bank minimises the distance $\|A_S R_S - Y_S L_S\|$ over time.

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Although the bank chooses $L_{s+1}$ at time $s + 1$, the decision is partly built on the information the bank has collected in the past. So, we can imagine that, at time $s$, the bank chooses $L_s$ and $L_{s+1|s}$ as well, conditional on the information set $\mathcal{I}_s$, i.e. $\{L_s, L_{s+1|s}\} \in F(\mathcal{I}_s)$, $F: \mathcal{I} \to L$, $s \in \{0, 1, 2, \ldots, \infty\}$

Obviously, the choice of $L_{s+1|s}$ is not conclusive since the bank may still adjust $L_{s+1}$, if an innovation occurs: the difference between $L_{s+1}$ and $L_{s+1|s}$ measures the bank’s reaction to shocks at $s + 1$, i.e. $L_{s+1} - L_{s+1|s} = \zeta \eta_{s+1}$, where $\zeta$ is the bank’s sensitivity to innovation $\eta_{s+1}$.
From the first-order condition of (1) over $L_s$ and $L_{s+1|s}$, assuming $E_sY_{s+1} \neq 0$ and dropping $cov(Y_{s+1}, A_{s+1}R_{s+1} - Y_{s+1}L_{s+1})$, we obtain

$$
\frac{E_s \Delta L_{s+1}}{L_s} = \frac{E_s \Delta A_{s+1}}{A_s} + \frac{E_s \Delta R_{s+1}}{R_s} + \frac{E_s (\Delta A_{s+1} \Delta R_{s+1})}{A_s R_s} - \frac{E_s \Delta Y_{s+1}}{Y_s} - \frac{E_s (\Delta Y_{s+1} \Delta L_{s+1})}{Y_s L_s}
$$

(2)

where $\Delta_{s+1}(A_s R_s) = A_{s+1}R_{s+1} - A_s R_s$, $\Delta A_{s+1} = A_{s+1} - A_s$, the same holding for $\Delta_{s+1}(Y_s L_s)$, $\Delta R_{s+1}$, $\Delta Y_{s+1}$ and $\Delta L_{s+1}$.

Subtracting and dividing both sides of the transition equation in (1) by $L_s$, we obtain

$$
\frac{\Delta L_{s+1}}{L_s} = \frac{E_s \Delta L_{s+1}}{L_s} + \varepsilon_{s+1}
$$

(3)

with $\varepsilon_{s+1} \sim (0, \sigma_{s+1})$

G. Maddaloni, 'Bank risks and liquidity dynamics'
Finally, from (2), (3) and lowercase letters representing log first-differences over \( s \), we obtain

\[
l_{s+1} = E_s a_{s+1} + E_s r_{s+1} + \sigma_{s+1}^{ar} - E_s y_{s+1} - \sigma_{s+1}^{yl} + \varepsilon_{s+1}
\]

where \( \sigma_{s+1}^{ar} = \frac{E_s (\Delta A_{s+1} \Delta R_{s+1})}{A_s R_s} \) and \( \sigma_{s+1}^{yl} = \frac{E_s (\Delta Y_{s+1} \Delta L_{s+1})}{Y_s L_s} \)

G. Maddaloni, 'Bank risks and liquidity dynamics'
As the pair $\{L_s, L_{s+1} | s\}$ is the bank’s optimal choice, which solves problem (1) given the information at time $s$ and innovation at $s + 1$, so must be the liquidity dynamics established by equation (4)
The central bank's loss function $\rho(M_s, \theta_s) \geq 0$, convex and differentiable, grows with the banking system's financial risks $\theta_s$, which are not under the central bank's control.

However, as innovations are known, the central bank provides cash $M_s$ in order to mitigate banks' financial risks.
The central bank’s problem is

\[ W(M) = \min_{\{M_s \in F(J_s)\}_0^\infty} \rho(M_s, \theta_s) + \tilde{\beta}^{s+1} E_s W(M_{s+1}) \]  

s.t. \( M_{s+1} = M_s + \Delta M_{s+1} \), \( \theta_{s+1} = \theta_s + \Delta \theta_{s+1} \), \( \Delta \theta_{s+1} \sim (0, \varphi_{s+1}) \)

with partial derivatives \( \rho_M \leq 0, \theta \geq 0, 0 < \tilde{\beta} < 1 \) the time discount factor and \( E_s \) the expectation operator, given information available at time \( s \)

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If shock $\Delta \theta_{s+1}$ occurs, the central bank supplies $\Delta M_{s+1}$ additional cash to offset the effect. In other words, differentiating (5) over $\Delta \theta_{s+1}$ and $\Delta M_{s+1}$ and for a given value of $\rho$, say $\bar{\rho}$

$$\rho_\theta \Delta \theta_{s+1} = -\rho_M \Delta M_{s+1}$$  (6)
Liquidity Dynamic Equilibrium: Definition

From the equilibrium condition of liquidity demand and supply dynamics over $s$, i.e. equating (4) and (6), we get

$$l_{s+1} = m_{s+1} + E_s a_{s+1} + E_s r_{s+1} + \sigma_{s+1}^{ar} - E_s y_{s+1} - \sigma_{s+1}^{yl} + \xi_{s+1}$$

(7)

where $m_{s+1}$ is the central bank’s money supply log first-difference and $\xi_{s+1} = \varepsilon_{s+1} + \frac{\rho \Delta \theta_{s+1}}{M_s \rho_M}$

G. Maddaloni, 'Bank risks and liquidity dynamics'
LIQUIDITY DYNAMIC EQUILIBRIUM: 
MODEL’S TESTING

Since they are unobservable, liquidity shocks can be modelled from residuals $\xi_{s+1}$ in (7), i.e.

$$\xi_{s+1} = l_{s+1} - m_{s+1} - E_s a_{s+1} - \sigma_{s+1}^{ar} - E_s r_{s+1} + E_s y_{s+1} + \sigma_{s+1}^{yl}$$

as, for example, a GARCH (1,1) process. Precisely, we can regress liquidity log-variations on $m_{s+1}$ and conditional expectations $a_{s+1|s}$, $r_{s+1|s}$, $\sigma_{s+1|s}^{ar}$, $y_{s+1|s}$, $\sigma_{s+1|s}^{yl}$ and finally estimate parameters in (7)
LIQUIDITY DYNAMIC EQUILIBRIUM:
MODEL’S TESTING

Compactly, let \( x_{s+1} \) be the vector of \( n \) explanatory variables, i.e.

\[
x_{s+1} = \mu_{s+1} + \omega_{s+1}
\]

(8)

where \( \mu_{s+1} \) and \( \omega_{s+1} \sim (0, \Sigma_{s+1}) \) are the drift and the noise vectors and \( \Sigma_{s+1} \) is the \( n \)-dimensional covariance matrix. From (7) and (8) and conditional expectations \( x_{s+1|s} \), we write

\[
l_{s+1} = \delta_0 + \delta_1 m_{s+1} + x'_{s+1|s} \delta + \xi_{s+1}
\]

(9)

where \( \delta \)'s are elasticities to be estimated, \( \xi_{s+1} = v_{s+1} \psi_{s+1} \) is the liquidity risk process, \( v^2_{s+1} = \gamma_0 + \gamma_1 \xi^2_s + \gamma_2 v^2_s \) and \( \psi_{s+1} \) are i.i.d. \( \sim (0,1) \) noises.
MODEL’S TESTING: CONDITIONAL EXPECTATIONS

In continuous time, the percentage change of $X(s)$ can be represented as a stochastic differential equation

$$\frac{dX(s)}{X(s)} = \mu(s)ds + \sqrt{\varrho(s)}dw$$

where $\mu(s)$ is the drift, $\varrho(s) < \infty$ the volatility and $dw \sim (0, ds)$ the noise. Then, Taylor-expand $\log(X)$ around $X_0$

$$\log(X) = \log(X_0) + \frac{1}{X_0}(X - X_0) - \frac{1}{2X_0^2}(X - X_0)^2 + \text{remainder}$$

Letting $X \rightarrow X_0$, taking expectations and dropping terms of order higher than $ds$, from (10) we obtain

$$E[d\log(X)] = \left(\mu - \frac{\varrho}{2}\right)ds$$

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MODEL’S TESTING: CONDITIONAL EXPECTATIONS CONTINUED

Discretise (10) over \( s \) as

\[
\frac{\Delta X_{s+1}}{X_s} = \mu_{s+1} + \sqrt{\varrho_{s+1}} \Delta w_{s+1}
\]

where \( \mu_{s+1} \) is the drift, \( \varrho_{s+1} \) the volatility and \( \Delta w_{s+1} \sim (0,1) \) the noise, as before. Then, like in (11), get the log first-difference conditional expectation \( x_{s+1|s} \)

\[
x_{s+1|s} = \mu_{s+1|s} - \frac{\varrho_{s+1|s}}{2}
\]

Assume \( x_{s+1} \) behaves as a random walk, i.e. \( \mu_{s+1} = x_s \), and estimate \( \varrho_{s+1|s} \) as the latest 20-period moving variance. Then use those explanatory variables’ expectations, i.e. \( x_{s+1|s} \), to estimate (9)
MODEL’S TESTING: DATA

The analysis has been conducted on business daily time series over a ten-year period, from January 2007 to December 2016.

Raw time series have been transformed into daily log first-differences and expectations have been computed under the random-walk hypothesis.

All data are drawn from financial time series publicly available in the internet. Specifically:

- Unsecured interbank money markets' interest rates have been taken from the European Money Market Institute's (EMMI) web site.
- Daily series on euro area repo contracts' rates and volumes have been drawn from the RepoFunds Rate's (RFR) web site, which collects aggregate information from BrokerTec and Mercato telematico dei Titoli di Stato (MTS) platforms accounting for most euro area repo contracts.
- Statistics on euro area banks' liquidity reserves, central bank's open market operations (i.e. MROs, LTROs, FTOs and structural operations), marginal lending facility and others (i.e. domestic credit, triple-A-rated euro area sovereign one-year yield, bond-market stress index, two-or-more EU sovereigns' default joint probability) have been taken from ECB web pages and from the disclosed part of ECB Statistical Data Warehouse (SDW).

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**Model's Testing: GARCH (1,1) Findings**

Euro area banks' liquidity dynamics, sustained by ECB's accommodative supply, has been particularly sensitive to stress-induced shocks.

Expectations of risk-related variables appear to explain a large fraction of the remaining liquidity share.

Expected log-variations of observed variables, under the random-walk process hypothesis, and the liquidity demand and supply's dynamic equilibrium explain nearly 59 percent of banks' aggregate reserves changes. The remaining 41 percent is represented by the GARCH (1,1) process, exhibiting very long memory of lagged conditional variances and disturbances, whose parameters' values sum over 0.99.

The normal distribution hypothesis is rejected and the GARCH estimation is carried out under Generalised Error Distribution (GED) assumption: the GED parameter's estimate is 0.71, which points to a leptokurtic distribution, as commonly occurs with financial times series.

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**MODEL’S TESTING – GARCH ESTIMATION (EViews 7)**

Dependent Variable: RESERVES  
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)  
Date: 04/01/18  Time: 11:03  
Sample (adjusted): 1/30/2007 12/30/2016  
Included observations: 2543 after adjustments  
Convergence achieved after 29 iterations  
Presample variance: backcast (parameter = 0.7)  
GARCH = C(11) + C(12)*RESID(-1)^2 + C(13)*GARCH(-1)

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<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
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Variance Equation

| C                     | 3.65E-05   | 9.37E-06   | 3.890394    | 0.0001  |
| RESID(-1)^2           | 0.053890   | 0.008991   | 5.993629    | 0.0000  |
| GARCH(-1)             | 0.936990   | 0.007902   | 118.5779    | 0.0000  |

GED PARAMETER: 0.709583  | 0.021914   | 32.38060   | 0.0000  

R-squared: 0.589568  Mean dependent var: 0.000629  
Adjusted R-squared: 0.588109  S.D. dependent var: 0.170487  
S.E. of regression: 0.109416  Akaike info criterion: -3.038813  
Sum squared resid: 30.32496  Schwarz criterion: -3.006656  
Log likelihood: 3877.850  Hannan-Quinn criter: -3.027148  
Durbin-Watson stat: 2.254802

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LIQUIDITY RISK MANAGEMENT: DEFINITIONS

Given the expected daily liquidity percent change $l_{s+1|s}$, we can set some confidence level $\alpha \in (0,1)$ such that $LS(\alpha)$ is the smallest liquidity shortfall, also expressed as a daily percentage, occurring with probability at most as big as $1 - \alpha$, that is

$$LS(\alpha) = \inf \{ l \in \mathbb{R} : \Pr(LS \geq l) \leq 1 - \alpha \}$$

Probabilistically, $LS$ is a quantile of the liquidity shortfall distribution and confidence values for $\alpha$ may be, say, 0.95, 0.975 or 0.99. For those confidence levels we define, given the liquidity shortfall's occurrence, the expected liquidity shortfall $ELS$, as

$$ELS(\alpha) = \frac{1}{1 - \alpha} \sum_{l \geq LS(\alpha)} l \cdot \Pr(l)$$

where $\Pr(l)$ is the probability assigned, on the grounds of the estimated GARCH distribution, to the liquidity shortfalls at least as big as $LS(\alpha)$
Liquidity Risk Management: Intuition

Expected liquidity change $l_{t+1}\mid_s$, liquidity shortfall $LS$ and expected liquidity shortfall $ELS$ at $\alpha$-confidence level

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Liquidity Risk Management: Empirical Results

According to the GARCH (1,1) results, the liquidity shortfalls of the estimated shocks' distribution at 0.95, 0.975 and 0.99 confidence levels are $LS(0.95) = 0.107$, $LS(0.975) = 0.198$ and $LS(0.99) = 0.365$

This means that banks' precautionary liquidity buffers, at the 0.95 confidence level, should be added 11.3 percent to the daily expected change. Liquidity positions should be increased by 21.9 and 44 percent for reaching 0.975 and 0.99 confidence level

On the other hand, the daily expected liquidity shortfalls, estimated at the corresponding confidence levels, are $ELS(0.95) = 0.298$, $ELS(0.975) = 0.456$ and $ELS(0.99) = 0.727$
Precautionary buffers may complement the liquidity requirement set under business-as-usual conditions, see Maddaloni (2015)

If large changes are expected, from the $R^2$ statistics of the GARCH estimation, we establish $LS(\alpha)$ to be at least as big as 0.695 (which is obtained as 0.41/0.59) times the daily expected liquidity change $l_{s+1|s}$ and, given the expected outgoing payments percent change $p_{s+1|s}$, we find

$$\frac{\lambda^*_{s+1|s} - \lambda^*_s}{\lambda^*_s} = l_{s+1|s} + \max\{0.695 \cdot l_{s+1|s}, LS(\alpha)\} - p_{s+1|s}$$

where $\lambda^*_{s+1|s}$ is the optimal liquidity ratio (as a share of daily outgoing payments) at $s + 1$ given information available at time $s$, which is found to be the optimal liquidity turnover ratio’s inverse by Maddaloni (2015), i.e.

Liquidity turnover ratio = $\frac{\text{Daily outgoing payments}}{\text{Daily average liquidity balance}}$
Estimated liquidity shocks and sensitivities can also be used for stress testing purposes.

For instance, stress-induced shocks, derived from the GARCH-estimated distribution at different $\alpha$-probability levels, can be applied to banks' current and prospective cash outflows against their liquidity positions, which would be assessed as satisfactory if they meet outstanding stressed payment obligations in full.

A similar argument goes for risk-related variables, whose effects on banks' liquidity positions may be evaluated over a consistent time horizon by stressing sensitivities with a multiple of their GARCH-estimated standard deviation.
Questions?
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Thank you