Bank risks and liquidity dynamics: evidence from
the euro area financial crisis

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Abstract

This paper attempts to model the European Central Bank (ECB) and
the euro area banking system’s reactions to risks brought by the financial
turmoil, from January 2007 to December 2016. Given a ‘representative’
bank’s demand and the central bank’s supply of cash against available
information on explanatory variables over time, liquidity risks are de-

erived as unexpected changes of traded liquidity amounts. The equilibrium
condition between liquidity demand and supply dynamics is empirically
tested, under a generalised autoregressive conditional heteroskedasticity
(GARCH) error term process, to estimate euro area bank risks’ evolution
and persistence. Those results may serve as a theoretical and empirical
backing to periodical stress testing euro area banks’ liquidity positions
as well as establishing capital requirements, in terms of precautionary
liquidity buffers.

Keywords: bank liquidity risk; financial crisis; financial regulation;
financial stability; liquidity requirement; stress test.

JEL Classification: C1; C7; D8; E4; E5; G1; G2; K2.

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1 Introduction

This paper attempts to illustrate how the European Central Bank (ECB) and the euro area banking system have reacted to risks brought by the financial turmoil, i.e. the subprime credit crisis first and the sovereign debt crisis later, from January 2007 to December 2016. To this aim, I model a ‘representative’ bank’s demanding and the central bank’s supplying cash over time to stem liquidity risks. Liquidity risks are modelled as the ‘representative’ bank and the central bank’s reactions to innovations, in the form of unexpected changes of traded liquidity amounts. The model’s closure, which establishes an equilibrium condition between liquidity demand and supply dynamics, is empirically tested, under a generalised autoregressive conditional heteroskedasticity (GARCH) error term process, to estimate euro area bank risks’ evolution and persistence.

Under uncertainty, banks demand and hold liquidity for transactional purposes, as long as this ensures the firm’s financial stability and long-term profitability. Banks need daily and intraday liquidity balances to settle customers’ and own transaction obligations. Usually, banks are able to fund current and prospective cash outflows on the grounds of expectations about investment opportunities, yields, market and counterparty risk dynamics, rollover and outstanding payment obligations against cash inflows produced by maturing assets and liabilities issuance, over a certain time horizon.

However, additional liquidity can ease temporary tensions due to occasional market frictions or unexpected evolution of transactions and incoming and outgoing payments. Then, as knowledge about future events is limited, banks may decide to hold liquidity buffers in order to cope with unexpected temporary events negatively affecting the bank’s ability to obtain funds, say, collecting deposits, rolling over loans in the money market, issuing debt at viable costs and settle payment obligations in an orderly manner.

In adverse market conditions – e.g. incoming cash flows reveal far below expectations; withdrawals and other outgoing payments are suddenly well beyond what anticipated; rollover and funding become abruptly scarce and prohibitive – banks might be forced, in order to honour outstanding debts when they fall due, to early liquidation of illiquid assets at undervalued prices. In a time of widespread, heightened and prolonged market and counterparty risks, banks cannot easily forecast their liquidity needs. Funding and investing – amid illiquidity of money and securities markets, increased yields volatility, bigger uncertainty about counterparty solvency – becomes more and more painful, in terms of rising costs and potential losses, possibly leading to bankruptcy. The bigger the uncertainty, risks and persistence of financial markets distress, the more likely banks hoard liquidity, limit credit exposures to households and firms, restrain lending in the interbank markets and resort to central bank’s funding. That is what apparently happened in the last decade or so within euro area financial markets. A brief recount of this is given in the next section.

The literature identifies two types of liquidity risks, although authors may call them with different names: market liquidity risk and funding liquidity risk. The first refers to how easy traders can liquidate positions without incurring
financial losses due to market price changes. This depends on the sort of assets
to liquidate, how many traders are willing to buy at current market prices and
how quickly assets can be traded, which largely depends on the amount to sell.
The second one refers to the financial institution’s ability to quickly raise cash in
the form of loans to fulfil outstanding payment obligations stemming from, say,
depositor withdrawals, debtor’s drawing on standing credit lines, margin calls on
either centrally or non-centrally cleared trades and so on. Whichever definition
we may look at, the underlying driver boils down to the financial institution’s
needing some liquidity, whose cost sustainability and timely availability become
crucial to the firm’s survival, to make payments for a number of reasons.

Before the financial crisis, stress testing of liquidity positions was not a
widely acknowledged risk management practice within the banking sector. From
a regulatory standpoint, Basel II rules mostly focused on capital requirement
for credit risk and market risk, allowing more skillful banks to develop internal
models and allocate capital to cover such risks. Liquidity risks were then mostly
treated as part of the market risks banking institutions would take on with little
specialness, if any.

Since the last financial crisis, liquidity risks have been receiving considerable
attention. In fact, financial institutions relying on wholesale funding experienced
rollover problems, as unsecured deposit markets suddenly dried up for lack of
investors’ confidence, and many assets were traded for cash at fire-sale prices.
Although Basel III eventually establishes a liquidity requirement to withstand
a 30-day liquidity stress scenario, the Liquidity Coverage Ratio (LCR), the rule
does not appear to be supported empirically and theoretically enough for the
banking industry to convincingly agree upon.

The present study recognises liquidity risks’ specific features and tries to
provide a hard-fact-based methodology to assess banks’ daily liquidity positions.
Using the quantitative information produced by the crisis itself, this study shows
how to learn from those institutions most involved in the management of the
turmoil, i.e. the banking system and the central bank. Under uncertainty,
I model the bank holding cash to settle daily payment obligations as long as
excess liquidity does not harm the bank’s own profitability, for missing yielding
investments. Specifically, the bank’s problem is to select the amount of liquidity
over discrete time so as to minimise illiquidity costs of financial risks, i.e. the
liquidity risk, against the opportunity cost of holding cash. Although the bank
chooses central bank money balances each time, the decision is partly built on
the information the bank has collected in the past. The difference between
current cash holdings and conditional expectation measures the bank’s reaction
to liquidity shocks. Likewise, the central bank’s loss function grows with the
banking system’s financial risks, which are not under the central bank’s control.
However, as innovations are known, the central bank provides cash in order
to mitigate banks’ financial risks. Liquidity risks are modelled as euro area
banks and ECB’s reactions to innovations, in the form of unexpected changes of
traded liquidity amounts. The equilibrium condition between liquidity demand
and supply dynamics is then empirically tested under a GARCH (1,1) process
to estimate euro area bank risks’ evolution and persistence.
My model’s approach is different from others, which view central banks’ intervention as targeted to drive financial markets’ prices and transactions’ volumes. For instance, Brunetti et al. (2011) find that ECB’s interventions could not affect prices, spreads, and interbank volumes’ levels and volatilities, as they instead used to in pre-crisis times: in fact, they conclude that during the crisis ECB’s interventions were accompanied by increased spreads and lower trading volumes. My interpretation, both theoretically and empirically, is that ECB’s measures accommodated banks’ precautionary liquidity demand dynamics amid increased financial risks perceptions, uncertainty and shocks, of which prices, spreads and volumes’ volatilities and levels are symptoms and reactions. Likewise, a positive correlation between funding liquidity risk and interest rates is found by Drehmann and Nikolaou (2013), who propose an insurance premium from banks’ bids as a measure of funding liquidity risk and show that aggressive bidding at ECB’s auctions, after August 2007, reveals such risk.

The empirical analysis is carried out on business daily time series over a ten-year period, from January 2007 to December 2016, and explanatory variables’ expected log-variations are computed under random-walk hypothesis. Estimation results show that euro area banks’ liquidity dynamics, sustained by ECB’s accommodative supply, has been particularly sensitive to stress-induced shocks. In addition, expectations of risk-related variables appear to explain a large fraction of the remaining liquidity share. Specifically, the equilibrium condition of liquidity demand and supply dynamics’ observed variables and expectations explain nearly 59 percent of banks’ aggregate reserves changes. The remaining 41 percent is due to the perceived noise process, exhibiting very long memory of lagged conditional disturbances and leptokurtic distribution, as commonly occurs with financial times series. As to policy and liquidity risk management tools, the estimated shocks’ historical distribution suggests, at the 0.95 confidence level, banks’ precautionary liquidity buffers be added 11.3 percent to the daily expected change while, at 0.975 and 0.99 confidence level, liquidity positions should be increased by 21.9 and 44 percent, respectively. Those precautionary buffers may then complement liquidity requirements set under business-as-usual conditions, like the one designed by Maddaloni (2015). As Maddaloni (2015) investigates and compares liquidity risk policies in terms of effectiveness and efficiency under stable risk conditions, this study addresses liquidity risks and precautionary buffers in a changing environment. Hence, the studies are mutually complementary, since the first one examines liquidity risk policies from a structural standpoint while this one does dynamically.

Modelling banks and the central bank’s reactions to innovations and estimating shocks’ size from historical time series may be a way to understand how big liquidity buffers should be in a real-like stress scenario. As a matter of fact, since empirically conceived under the financial crisis – an extreme event which actually occurred – liquidity requirements of the sort may turn more acceptable than others. This may then help both banks and regulators build a common methodology to design effective policy tools and reasonably prevent liquidity risks from occurring again in the future, containing negative side-effects on financial markets’ efficiency and economic growth.
The paper is organised as follows. In Section 2, I describe the theoretical model while in Section 3 details of the econometric model to be tested are given. After data description in Section 4, I discuss the GARCH estimation results in Section 5. In Section 6, I sketch some policy implications and show how the GARCH estimation results may usefully find application in liquidity risk management as precautionary liquidity buffers and liquidity stress testing. The last section is left to conclusions.

1.1 A review of the euro area banking crisis

Since 2007, the financial crisis has heavily affected the euro area interbank money market, by increasing liquidity risk and counterparty credit risk. According to the ECB’s Euro Money Market Survey, in 2010 euro money market aggregate turnover decreased for three years in a row, as a consequence of interbank trades contraction and, due to ECB’s refinancing operations, the liquidity surplus environment.\(^1\) In particular, the fall in the interbank unsecured money market turnover stemmed from financial institutions avoiding counterparty credit risk and their moving to secured funding.\(^2\) However, as the ECB’s Euro Money Market Study (2010) points out, an increasing number of banks, especially large, also financed by means of short-term securities, namely certificates of deposits (CDs), rather than interbank deposits. Amid financial markets turmoil, the Eurosystem took extraordinary measures to improve banks’ liquidity positions and reduce money market spreads and interest rates.\(^3\) In early 2010 tensions emerged in the euro area government bond markets, with growing spreads of peripheral euro area countries’ ten-year government bond yields over Germany’s, mainly as a result of increasing market concerns about fiscal sustainability as well as the deepening and prolonging economic crisis. In addition to other measures, the ECB re-introduced the fixed-rate tender procedure with full allotment in ordinary three-month LTROs and added six-month full-allotment LTROs; in 2011 the ECB announced two more LTROs, with 12 to 13-month maturities, as well as the continuation of full-allotment fixed-rate MROs. Furthermore, by end 2011, the ECB conducted two three-year LTROs with option of early repayment, partly or in full, after one year; reduced the

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\(^1\) Open market operations include main refinancing operations (MROs), longer-term refinancing operations (LTROs), fine tuning operations (FTOs) and structural operations.

\(^2\) There are two main segments in the euro area money market. One is the ‘unsecured’ market, which is concentrated on very short maturities, mainly overnight. The other is the repurchase agreement (repo) market, the largest euro money market with maturities mostly up to one month, which is known as ‘secured’, since lending is against collateral. In this respect, Mancini et al. (2015) find that, during the financial crisis, the central counterparty-based euro interbank repo market functioned better than other funding markets and even acted as a shock absorber.

\(^3\) ECB’s measures focused on euro area banks and were characterised by ‘fixed-rate full-allotment’ tender procedures in all refinancing operations, with unlimited central bank liquidity to financial institutions at the main refinancing rate and against eligible collateral. Moreover, the lists of eligible collateral and counterparties for refinancing operations were extended. Finally, the ECB implemented additional LTROs with maturities of up to six months.
reserve ratio from 2 to 1 percent; broadened the set of collateral eligible for refinancing operations. While issuance of medium- to longer-term debt have fallen with banks’ senior unsecured debt being adversely affected by the rising risk aversion, in some euro area countries, especially those hit by intermittent sovereign and bank distress, according to ECB’s Financial Stability Review (2013), banks’ shares of sovereign bond holdings on total assets have risen above pre-crisis levels.\footnote{Large sovereign debt holdings have been exposing euro area banks to substantial interest rate volatility, depending on portfolios’ duration, position hedging and sovereign debt grades. As a matter of fact, Acharya and Steffen (2015) view euro area banks’ risks in 2007–2013 as a form of carry trade, with banks arbitraging between short-term unsecured funding and long positions on peripheral sovereign bonds. With the financial crisis, the spreads between the two legs of the trade diverged, resulting in significant losses for banks and leading to concerns about their solvency and liquidity.} As to funding costs, banks’ ability to finance has been suffering home-country fiscal sustainability concerns, resulting in financial market’s fragmentation.\footnote{According to ECB’s Financial Stability Review (2012, 2013), this process was most acute for smaller banks from stressed countries while debt issuance by large and smaller banks in non-stressed countries suffered less from changing market conditions.} Although market conditions for bank debt instruments have more recently improved, ECB’s Financial Stability Review (2015, 2017) argues that banks may have replaced more expensive debt funding with Eurosystem’s financial support. The poor performance of bank equity prices has in fact shown, as price-to-book ratios have decreased to low levels, reflecting market doubts about banks’ asset profitability, mostly stemming from long-standing non-performing loans (NPL).\footnote{For further reference to the euro area financial crisis, see relevant editions of ECB’s Euro Money Market Study and Financial Stability Review.}

2 Theoretical model

Under uncertainty, the bank retains cash to settle daily payment obligations as long as excess liquidity does not harm the bank’s own profitability, for missing yielding investments. Specifically, the bank’s problem is to select the amount of liquidity over time so as to minimise illiquidity costs of financial risks, i.e. the liquidity risk, against the opportunity cost of holding cash. More formally, the bank chooses cash $L_s \geq 0$ to minimise $c(L_s)$ over discrete time $s$, i.e.

$$V(L) = \min_{\{L_s, L_{s+1} \in F(\mathcal{X}_s)\}} c(L_s) + \beta^{s+1} E_s V(L_{s+1})$$

s.t. $L_{s+1} = L_{s+1}|_s + \zeta \eta_{s+1}$, $\zeta > 0$, $\eta_{s+1} \sim (0, \phi_{s+1})$,

where $c(L_s) = (A_s R_s - Y_s L_s)^2$, $A_s$ is the amount of the bank’s assets bearing financial risks $R_s$, $Y_s$ is the foregone yield for holding cash $L_s$, $0 < \beta < 1$ is the time discount factor and $E_s$ the expectation operator, given information available at time $s$. The intuition behind the quadratic cost is that the bank holds cash just enough to offset the cost associated with the liquidity risk and
prevent profitability from suffering excess liquidity. Hence, I simply assume the
bank wants the illiquidity cost and the opportunity cost to be as close as possible
or, more formally, the bank minimises the distance \( \|A_s R_s - Y_s L_s\| \) over time.
Although the bank chooses \( L_{s+1} \) at time \( s+1 \), the decision is partly built on the
information the bank has collected in the past. So, we can imagine that the bank
chooses \( L_s \) and \( L_{s+1|s} \) as well, conditional on the information set \( \mathcal{I}_s \) available at
time \( s \), i.e. \( \{L_s, L_{s+1|s}\} \in F(\mathcal{I}_s), F : \mathcal{I} \to L, s \in \{0, 1, 2, \ldots, \infty\} \). Obviously,
the choice of \( L_{s+1|s} \) is not conclusive since the bank may still adjust \( L_{s+1|s} \)
if an innovation occurs: the difference between \( L_{s+1|s} \) and \( L_{s+1|s} \) measures the
bank’s reaction to shocks at \( s+1 \), i.e. \( L_{s+1} - L_{s+1|s} = \zeta \eta_{s+1} \) where \( \zeta \) is the
bank’s sensitivity to innovation \( \eta_{s+1} \).

From the first-order condition of (1) over \( L_s \) and \( L_{s+1|s} \) and after some
manipulation, we obtain

\[
l_{s+1} = E_s a_{s+1} + E_s r_{s+1} + \sigma_{\alpha s+1} - E_s y_{s+1} - \sigma_{\delta s+1} + \varepsilon_{s+1}, \tag{2}
\]

where \( \sigma_{\alpha s+1} = \frac{E_s(\Delta L_{s+1})}{A_s R_s}, \sigma_{\delta s+1} = \frac{E_s(\Delta Y_{s+1} - \Delta L_{s+1})}{Y_s L_s}, \varepsilon_{s+1} = \frac{\zeta \eta_{s+1}}{L_s} \sim (0, \sigma_{s+1}) \)
and lowercase letters representing log first-differences over \( s \). Proof of (2) is given
in Appendix.

The rationale behind (2) is that as the pair \( \{L_s, L_{s+1|s}\} \) is the bank’s optimal
choice, which solves problem (1) given the information available at time \( s \) and
the innovation at \( s+1 \), so must be the liquidity dynamics established by equation (2).

Let’s suppose now that the central bank’s loss function \( \rho \geq 0 \), convex and
differentiable, grows with a set of variables \( \theta_s \), representing the banking system’s
financial risks, which are not under the central bank’s control. However, the
central bank provides cash \( M_s \) to the banking system in order to mitigate banks’
financial risks. Then, at each \( s \), the central bank solves

\[
W(M) = \min_{\{M_s \in F(\mathcal{I}_s)\}_s} \rho(M_s, \theta_s) + \bar{\beta}^{s+1} E_s W(M_{s+1}) \tag{3}
\]
s.t. \( M_{s+1} = M_s + \Delta M_{s+1}, \theta_{s+1} = \theta_s + \Delta \theta_{s+1}, \Delta \theta_{s+1} \sim (0, \varphi_{s+1}) \),

with partial derivatives \( \rho_M \leq 0, \rho_\theta \geq 0 \) and \( 0 < \bar{\beta} < 1 \) the time discount factor.

If shock \( \Delta \theta_{s+1} \) occurs at \( s+1 \), the central bank reacts by \( \Delta M_{s+1} \) additional
cash to offset the effect. In other words, differentiating (3) over \( M_{s+1} \) and \( \theta_{s+1} \)
and for a given value of \( \rho \), say \( \bar{\rho} \),

\[
\rho_\theta \Delta \theta_{s+1} = -\rho_M \Delta M_{s+1}. \tag{4}
\]

From the equilibrium condition of liquidity demand and supply dynamics
over \( s \), i.e. equating (2) and (4), we finally obtain

\[
l_{s+1} = \eta_{s+1} + E_s a_{s+1} + E_s r_{s+1} + \sigma_{\alpha s+1} - E_s y_{s+1} - \sigma_{\delta s+1} + \varepsilon_{s+1}, \tag{5}
\]
where $\xi_{s+1} = \varepsilon_{s+1} + \frac{\sigma_{\delta_{s+1}}}{\Delta_{\rho_{t, s+1}}}$ and $m_{s+1}$ is the central bank's money supply log first-difference, as above.

### 3 Econometric model

Since they may be observable by the banking system and the central bank only, possibly because of private information, liquidity shocks can be modelled from residuals of (5), i.e.

$$\xi_{s+1} = l_{s+1} - m_{s+1} - E_s \sigma_{s+1} - E_s \sigma_{s+1} + \sigma_{\delta_{s+1}},$$

as, for example, a GARCH (1,1) process. Precisely, we can regress banks’ reserves log-variations on $m_{s+1}$ and conditional expectations $\sigma_{\delta_{s+1}}$, $\sigma_{\theta_{s+1}}$, $\tau_{s+1}$, $\gamma_{s+1}$, and estimate parameters in (5).

Compactly, let $x_{s+1}$ be the vector of $n$ explanatory variables’ log first-differences

$$x_{s+1} = \mu_{s+1} + \Delta \omega_{s+1},$$

where $\mu_{s+1}$ and $\Delta \omega_{s+1}$ are the drift and the noise vectors, respectively and $\Sigma_{s+1}$ is the $n$-dimensional covariance matrix.

Then, from (5), (6) and explanatory variables’ conditional expectations $x_{s+1}$, proof of which is in the Appendix, we write

$$l_{s+1} = \delta_0 + \delta_1 m_{s+1} + x_{s+1} \delta + \xi_{s+1},$$

where $\delta$’s are elasticities to be estimated, $x_{s+1}$ is the sample latest 20-period moving covariance up to time $s$.

Finally, from (7) and the GARCH (1,1) process, $\xi_{s+1} = v_{s+1} \psi_{s+1}$ represents the liquidity risk, with $v_{s+1}^2 = \gamma_0 + \gamma_1 \varepsilon_s^2 + \gamma_2 v_s^2$, $\gamma_0$, $\gamma_1$, $\gamma_2 > 0$, $\gamma_1 + \gamma_2 < 1$ and $\psi_{s+1} \sim \mathcal{N}(0, 1)$ i.i.d. noises.

### 4 Data description

The empirical analysis has been conducted on business daily time series over a ten-year period, from January 2007 to December 2016. Raw time series have been transformed into daily log first-differences and, in order to fit the theoretical model, expectations have been computed under the random-walk hypothesis I described in the previous sections.

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7 $\delta_0$ includes $\text{cov} (Y_{s+1}, A_{s+1} R_{s+1} - Y_{s+1} L_{s+1})$ average while variations are captured by residuals of (7).
All data are drawn from financial time series publicly available in the internet. Specifically, unsecured interbank money markets’ interest rates have been taken from the European Money Market Institute’s (EMMI) site, while daily series on euro area repo contracts’ rates and volumes have been drawn from the RepoFunds Rate’s (RFR), which collects aggregate information from BrokerTec and Mercato telematico dei Titoli di Stato (MTS) platforms accounting for most euro area repo contracts.\(^8\) Statistics on euro area banks’ liquidity reserves, central bank’s open market operations (i.e. MROs, LTROs, FTOs and structural operations), marginal lending facility and others (i.e. domestic credit, triple-A-rated euro area sovereign one-year yield, bond-market stress index, two-or-more EU sovereigns’ default joint probability) have been taken from the ECB’s web pages and from the publicly disclosed part of ECB’s Statistical Data Warehouse (SDW).\(^9\)

5 GARCH (1,1) estimation

The empirical analysis tests the theoretical model to estimate the banking system’s liquidity needs against expectations of loans extended to euro area private and public sectors and within the banking system as well, in the form of secured and unsecured funding, given perceived market, private and sovereign risks and returns and given the central bank’s reactions to shocks through open market operations and marginal lending. Consequently, the GARCH error term process attempts to represent euro area banking system financial risks’ evolution and persistence, as reflected by unexpected daily liquidity reserves changes.\(^10\)

As shown at Table 1, the explanatory variables of the euro area banking system’s aggregate reserves log-variations and the GARCH (1,1) residuals’ process convincingly support the assumptions underlying the theoretical model I developed in the previous sections.

Specifically, expected log-variations of observed variables, under the random-walk process hypothesis, and the liquidity demand and supply’s dynamic equilibrium apparently explain nearly 59 percent of banks’ aggregate reserves changes. The remaining 41 percent is represented by the GARCH (1,1) process, exhibiting very long memory of lagged conditional variances and disturbances, whose parameters’ values sum over 0.99. The normal distribution hypothesis is re-

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\(^8\) According to Eurex GC Pooling, the other main euro area trading platform for general collateral repo contracts, and RFR’s information, the average repo volumes traded on BrokerTec and MTS in April 2016 accounted over 67 percent of repo contract volumes traded on the three platforms. EMMI’s Euro repo and Eonia Swap index series have not been tested, because publicly available statistics have been discontinued since 2014.

\(^9\) Daily domestic credit and bond-market stress index series have been linearly interpolated from monthly and weekly observations, respectively. ECB’s targeted tools during the financial crisis (e.g. covered bond purchase programmes, CBPPs, securities markets programme, SMP, asset purchase programme, APP, targeted longer-term refinancing operations, TLTRO, outright monetary transactions, OMT, etc.) are not included in the analysis since they have been irregularly adopted along the observation period and they may not be meant to specifically stem the banking system’s financial risks.

\(^10\) GARCH estimation has been carried out using EViews 7 software.
ected and the GARCH estimation is consistently carried out under generalised error distribution (GED) assumption\textsuperscript{11}: in fact, the GED parameter’s estimate is just 0.71, which clearly points to a leptokurtic error distribution, as commonly occurs with financial times series.\textsuperscript{12}

Elasticity estimates of domestic credit and repo rates’ expected log-variations, 9.66 and 5.98 respectively, apparently carry a larger effect on banks’ reserves changes.\textsuperscript{13} This may depend on banks feeding customers’ credit lines with liquidity and on bigger profitability of interbank secured lending, with banks arbitraging between central bank’s refinancing rates and rising secured money market rates. Alternatively, rising risks and costs associated with those contracts, e.g. non-performing loans, sovereign yield volatility and collateral haircuts, may explain the dynamics as well. This is due to bigger uncertainty about future assets values, which in turn makes banks hold additional liquidity in the more likely case assets depreciate and do not provide the expected cash flow at maturity or on an early sale. This interpretation is confirmed by the bond-market stress index and the two-or-more EU sovereigns’ default joint probability expected log-variations parameters’ estimates, 0.01 and 0.02 respectively.

Similarly, as to the 2007-2008 financial crisis in the US, Ashcraft et al. (2011) provide evidence on asset-backed commercial paper (ABCP) price volatility increasing payments shocks and banks’ precautionary liquidity stemming from counterparty credit risk concerns, while Gorton and Metrick (2012) point to higher uncertainty about bank solvency and collateral lower values as the main cause for repo haircuts’ increases. Likewise, Brunnermeier and Pedersen (2009) conclude that margins may increase because of uncertainty about price changes and time-varying volatility. This happens when shocks lead to bigger current volatility, which in turn raises expected future volatility and margins as well.

The same reasoning most likely explains EONIA expected log-variations elasticity’s estimate (3.16), as it captures the effect of unsecured funding risks and costs on banks’ precautionary liquidity demand.\textsuperscript{14} As a matter of fact, this is consistent with the sensitivity estimate’s sign of overnight funds traded in euro area interbank unsecured markets (-0.008), representing the effect of anticipated fund availability changes on banks’ liquidity dynamics: when expected market funding availability decreases, because of rising counterparty credit risks, banks hold additional precautionary reserves. Conversely, repo transactions volumes expected log-variations do not appear to greatly affect banks’ liquidity demand changes. Since extending secured funding bears little risk – because counter-

\textsuperscript{11}Generalised hyperbolic (GH) error distribution, which allows for skewness, might also be worth looking into. Unfortunately, the econometric package I use does not provide this option.

\textsuperscript{12}In the estimation, to avoid near-singularity, explanatory variables expectations do not include covariances.

\textsuperscript{13}Domestic credit comprises banks’ lending to private and government sector as well, both in the form of loans and securities.

\textsuperscript{14}According to EMMI’s definition, EONIA (Euro OverNight Index Average) is the effective overnight reference rate for the euro. It is computed as a weighted average of all overnight unsecured lending transactions in the interbank market, undertaken in the European Union and European Free Trade Association (EFTA) countries. The ECB is the Calculation Agent for EONIA.
parties exchange cash with collateral discounted at current and prospected risk-adjusted market prices – the marginal effect on money demand dynamics (0.02) may just reflect the lender’s precautionary additional haircut. On the other hand, the 12-month interbank unsecured market risk dynamics and the euro area sovereign debt crisis’ flight-to-quality effect on triple-A rated euro area sovereign yields may interfere with the opportunity cost of banks holding liquidity, i.e. 12-month EURIBOR rate spread over triple-A rated euro area sovereign one-year yield expected log-variations (-2.66). The spread may actually reflect banks’ perceived cost of precautionary liquidity holdings, as the EURIBOR rate refers to prime banks’ offered rate, which incorporates little counterparty credit risk, against same-maturity highly liquid assets’, represented by best rated euro area sovereigns’ yield. Finally, ECB open market operations log-variations’ effect on banks’ aggregate liquidity (2.12) may be interpreted both as accommodative moves of liquidity supply against financial risks and demand dynamics as well as the central bank money multiplier within the banking system and the real economy, in the form of funds traded in interbank secured and unsecured markets and bank customers’ cash deposits and loans.

6 Policy implications

As it was pointed out by the Basel Committee on Banking Supervision (2011), one of the main reasons the financial crisis became so severe was that banks were holding insufficient liquidity buffers. According to the Committee, the difficulties experienced by some banks were due to lapses in basic principles of liquidity risk management. So, the Committee published in 2008 the Principles for Sound Liquidity Risk Management and Supervision, providing guidance on funding liquidity risk management and supervision. To complement these principles, in 2013 first and finally in 2014, the Committee established two minimum standards for funding liquidity: a) the Liquidity Coverage Ratio (LCR), to ensure that the bank has sufficient high quality liquid resources to survive an acute stress scenario lasting for one month; b) the Net Stable Funding Ratio (NSFR), to promote bank’s funding resilience over a one-year time horizon, through a sustainable maturity structure of assets and liabilities.

Along the LCR’s line, Maddaloni (2015) found a technically feasible and technology neutral daily liquidity requirement, based on banks’ payment and liquidity habits as they appear from real-time gross settlement (RTGS) transfer systems. In fact, the proposed requirement takes into account the intraday liquidity management techniques adopted by banks, i.e. how banks settle outgoing payments along the business day, which is concisely termed as the liquidity turnover ratio. In brief, the liquidity turnover ratio expresses the way banks manage daily transfers settlement, also by means of cash flows produced by

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As ECB’s Euro Money Market Study (2008) reports, EURIBOR contributors are asked to quote rates at which, to the best of their knowledge, euro interbank term deposits are being offered within the euro area by one (merely hypothetical) prime bank to another at 11 a.m. CET (“the best price between the best banks”).
incoming payments, by market practices of timing settlement and by the way
banks resort to interbank money markets and payment systems to obtain addi-
tional funding, in case the intraday management of queuing items cannot timely
provide with sufficient liquidity for settlement purposes.

Since Maddaloni (2015) establishes the liquidity requirement as business-
based in ordinary times, this can be complemented with additional buffers to
face more adverse, either market or firm-specific, stress scenarios. In the light of
the empirical results I discussed, precautionary buffers can be retrieved from the
liquidity shocks I estimated through the GARCH (1,1) process. For instance,
banks could increase daily liquidity reserves in order to cover liquidity shortages
within a certain confidence level of the empirical shocks’ GARCH distribution
and stressed sensitivities. Additional buffers of the kind appear to be empirically
supported and, hence, more acceptable than others, since they are conceived
under extreme but still plausible stress scenarios. As a matter of fact, what has
been studied just feeds from the recent financial crisis, an extreme event which
actually occurred.

Furthermore, the methodology above can also help regulators and individ-
ual banks periodically for liquidity stress-testing purposes. Shocks and stressed
parameters’ estimates from the GARCH model could be used in order to assess
both the banking system as a whole and individual banks’ liquidity buffers ade-
quacy under extreme but still plausible scenarios. Of course, regulators and
banks may identify and come up with more significant explanatory variables,
parameters and shocks’ estimates, since the chances are they possess more in-
formation, mostly private and confidential, than those publicly disclosed and
available in the internet.16

6.1 Implementing liquidity management tools

The results from the GARCH estimation of the previous section can be prac-
tically exploited for both liquidity risk management purposes, at firm’s level,
and policy rules, say, micro and/or macro prudential purposes, at authority’s
level. As a matter of fact, we can use the empirical shocks’ distribution, as
estimated with the GARCH (1,1), to prudentially increase liquidity positions
and face extreme but still plausible stress scenarios. Straightforward steps to
implement such a measure are described as follows.

Given the expected daily liquidity percent change \( l_{s+1|s} \), we can set some
confidence level \( \alpha \in (0, 1) \) such that \( LS(\alpha) \) is the smallest \textit{liquidity shortfall},
also expressed as a daily percentage, occurring with probability at most as big
as \( 1 - \alpha \), that is

\[
LS(\alpha) = \inf \{ l \in \mathbb{R} : \Pr (LS \geq l) \leq 1 - \alpha \}.
\]

16For example, payment and securities settlement systems information on banks’ cash and
securities transfers, settlement delays of queuing items and settlement failures may jointly
be examined with secured and unsecured interbank markets’ information on traded funds,
interest rates and collateral haircuts as well as information on central counterparties’ margin
calls.
Probabilistically, $LS$ is a quantile of the liquidity shortfall distribution and confidence values for $\alpha$ may be, say, $0.95$, $0.975$ or $0.99$. For those confidence levels we can also define, given the liquidity shortfall’s occurrence, the expected liquidity shortfall $ELS$, as

$$ELS(\alpha) = \frac{1}{1 - \alpha} \sum_{l \geq LS(\alpha)} l \cdot Pr(l),$$

where $Pr(l)$ is the probability assigned, on the grounds of the GARCH-estimated distribution, to the liquidity shortfalls at least as big as $LS(\alpha)$. The underlying intuition is pictured at Figure 1.

According to the GARCH (1,1) results, the liquidity shortfalls of the estimated shocks’ historical distribution at $0.95$, $0.975$ and $0.99$ confidence levels are $LS(0.95) = 0.107$, $LS(0.975) = 0.198$ and $LS(0.99) = 0.365$. This means that banks’ precautionary liquidity buffers, at the $0.95$ confidence level, should be added $11.3$ percent to the daily expected change. Instead, liquidity positions should be increased by $21.9$ and $44$ percent for reaching $0.975$ and $0.99$ confidence level, respectively. On the other hand, the expected daily liquidity shortfalls, estimated at the corresponding confidence levels, are $ELS(0.95) = 0.298$, $ELS(0.975) = 0.456$ and $ELS(0.99) = 0.727$.\(^{17}\)

However, in order to grant sufficient liquidity positions in case large changes are expected, from the $R^2$ statistics of the GARCH estimation, we conservatively establish $LS(\alpha)$ (or $ELS(\alpha)$, if we like) to be at least as big as $0.695$ (which is obtained as $0.41/0.59$ times the expected daily liquidity change $l_{s+1|s}$ and, given the expected outgoing payments percent change $p_{s+1|s}$, we find

$$\frac{\lambda^*_{s+1|s} - \lambda^*_s}{\lambda^*_s} = l_{s+1|s} + \max \{0.695 \cdot l_{s+1|s}, LS(\alpha)\} - p_{s+1|s},$$

where $\lambda^*_{s+1|s}$ is the optimal liquidity ratio (as a share of outstanding payment obligations) at $s + 1$ given information available at time $s$, which is found to be the optimal liquidity turnover ratio’s inverse by Maddaloni (2015).

Alternatively, estimated liquidity shocks and sensitivities can be used for stress testing purposes in order to assess liquidity buffers’ adequacy under extreme but still plausible stress scenarios. For instance, stress-induced shocks, derived from the GARCH-estimated distribution, at different $\alpha$-probability levels can be applied to banks’ current and prospective cash outflows against their liquidity positions, which would be assessed as satisfactory if, along the time span under examination, they meet outstanding stressed payment obligations in full. If, say, a 30-day period is considered, the $\alpha$-probability shock increase should be charged on banks’ current and prospective cash outflows through the GARCH-estimated process for 30 days. A similar argument goes for risk-related variables, whose effects on banks’ liquidity positions may be evaluated over a consistent time horizon by stressing sensitivities with a multiple of their GARCH-estimated standard deviation.

\(^{17}\)Reported quantile values are expressed as log-variations from the estimated GARCH distribution and tail-end distribution’s percent changes are actually bigger.
7 Concluding remarks

Since the recent financial crisis, liquidity risks have been receiving considerable attention by financial institutions, regulators, consultants and researchers in finance, risk management and economics as well.

Many solutions have been proposed to curb such risks, especially in the banking system, among which those established by financial institutions’ regulators, nationally and internationally. Nonetheless, consensus on the issue has not been reached yet as public debate is still going, more research has been carried out and presumably will continue for years to come.

This paper contributes to find a methodology that addresses liquidity risks by using the quantitative information produced by the financial crisis and tries to learn from the behaviours of those institutions more involved in the management of the turmoil, i.e. the banking system and the central bank.

Modelling banks and the central bank’s reactions to liquidity shocks and estimating the size of those shocks from historical time series may be a way to understand how big liquidity buffers should be in the presence of substantial, extensive, pervasive and persistent financial risks. This may help both banks and regulators build a common methodology to design effective policy tools and reasonably prevent liquidity risks from occurring again in the future, containing negative side-effects on financial markets’ efficiency and economic growth.

In this study, I have tried to reach some conclusion about the size of those risks by using publicly available information on euro area banks’ liquidity reserves, central bank’s open market operations and other financial data. However, more valuable and granular information is available exclusively to banks and supervisors as well, which may help get a clearer and comprehensive view of the underlying issues than the one I eventually managed.

Consequently, regulators and the banking community are strongly encouraged to carry out empirical studies akin to mine and possibly obtain useful results to eventually establish sensible requirements and supervisory standards they can convincingly agree upon.

Acknowledgements and disclaimer

The views expressed in the work and all errors, if any, are mine, for which the Bank of Italy is not responsible.

References


Appendix

Mathematical proofs

Proof of eq. (2)

The first-order conditions of (1) over $L_s$ and $L_{s+1}$, assuming $E_sY_{s+1} \neq 0$ and dropping $\text{cov}(Y_{s+1}, A_{s+1}L_{s+1})$, respectively are

$$A_sR_s = Y_sL_s, \quad (8)$$

$$E_s(A_{s+1}R_{s+1}) = E_s(Y_{s+1}L_{s+1}). \quad (9)$$

Dividing (9) by (8), we obtain

$$\frac{E_s(A_{s+1}R_{s+1})}{A_sR_s} = \frac{E_s(Y_{s+1}L_{s+1})}{Y_sL_s}. \quad (10)$$

Subtracting and dividing both sides of the transition equation in (1) by $L_s$, we obtain

$$\frac{E_s\Delta L_{s+1}}{L_s} = \frac{E_s\Delta A_{s+1}}{A_s} + \frac{E_s\Delta R_{s+1}}{R_s} + \frac{E_s(\Delta A_{s+1}\Delta R_{s+1})}{A_sR_s} - \frac{E_s(\Delta Y_{s+1}\Delta L_{s+1})}{Y_sL_s}, \quad (11)$$

where $\Delta_{s+1}(A_{s}R_{s}) = A_{s+1}R_{s+1} - A_{s}R_{s}$, $\Delta A_{s} = A_{s+1} - A_{s}$, the same holding for $\Delta_{s+1}(Y_{s}L_{s})$, $\Delta R_{s+1}$, $\Delta Y_{s+1}$ and $\Delta L_{s+1}$.

Subtracting and dividing both sides of the transition equation in (1) by $L_s$, we obtain

$$\frac{\Delta L_{s+1}}{L_s} = \frac{E_s\Delta L_{s+1}}{L_s} + \varepsilon_{s+1}. \quad (12)$$

Finally, from (10) and (11), we find (2). □

Conditional expectation

In continuous time, the percentage change of $X(s)$ can be represented as a stochastic differential equation, i.e.

$$\frac{dX(s)}{X(s)} = \mu(s)ds + \sqrt{\varphi(s)}dw, \quad (12)$$

where $\mu(s)$ is the drift, $\varphi(s) < \infty$ the volatility and $dw \sim (0, ds)$ the noise.
Then, Taylor-expand $\log(X)$ around $X_0$, i.e.

$$\log(X) = \log(X_0) + \frac{1}{X_0} (X - X_0) - \frac{1}{2X_0^2} (X - X_0)^2 + \sum_{n=3}^{\infty} c_n (-1)^{n-1},$$

where $c_n = \frac{(n-1) (X-X_0)^n}{n!X_0}$. 

Letting $X \to X_0$, taking expectations and dropping terms of order higher than $ds$, from (12) we get

$$E[d \log(X)] = \left( \mu - \frac{\rho}{2} \right) ds. \quad (13)$$

Indeed, we can discretise (12) over $s$ as

$$\frac{\Delta X_{s+1}}{X_s} = \mu_{s+1} + \sqrt{\rho_{s+1}} \Delta w_{s+1},$$

where $\mu_{s+1}$ and $\rho_{s+1}$ are the drift and the volatility, as above, and $\Delta w_{s+1} \sim (0, 1)$. Like in (13), we eventually find the log first-difference conditional expectation $x_{s+1|s}$ as

$$x_{s+1|s} = \mu_{s+1|s} - \frac{\rho_{s+1|s}}{2}.$$

□
Table 1 GARCH estimation output – EViews 7.

Dependent Variable: RESERVES  
Method: ML - ARCH (Marquardt) - Generalized error distribution (GED)  
Date: 04/01/18   Time: 11:03  
Sample (adjusted): 1/30/2007 12/30/2016  
Included observations: 2543 after adjustments  
Convergence achieved after 29 iterations  
Presample variance: backcast (parameter = 0.7)  
GARCH = C(11) + C(12)*RESID(-1)^2 + C(13)*GARCH(-1)

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<th>Variable</th>
<th>Coefficient</th>
<th>Std. Error</th>
<th>z-Statistic</th>
<th>Prob.</th>
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Variance Equation

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GED PARAMETER 0.709583  0.021914  32.38060  0.0000

R-squared      0.589568  Mean dependent var  0.000629
Adjusted R-squared  0.588109  S.D. dependent var  0.170487
S.E. of regression   0.109416  Akaike info criterion -3.038813
Sum squared resid   30.32496  Schwarz criterion -3.006656
Log likelihood     3877.850  Hannan-Quinn criter. -3.027148
Durbin-Watson stat  2.254802
Figure 1 Expected liquidity change $L_{s+1}$, liquidity shortfall $LS$ and expected liquidity shortfall $ELS$ at $\alpha$-confidence level.