Assessing the Cyclical Implications of IFRS 9: A Recursive Model*

Jorge Abad
CEMFI

Javier Suarez
CEMFI & CEPR

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Abstract

IFRS 9 is the new accounting standard for the valuation of financial assets and liabilities. Its key innovation is the shift from an incurred loss approach to an expected loss approach to the measurement of credit impairment losses. The new allowances must equal the discounted one-year expected losses when exposures have not suffered a significant deterioration of credit quality and the discounted lifetime expected losses otherwise. This paper develops a recursive model for the assessment of the implications of different measurement approaches for the average levels and dynamics of the allowances of a bank's loan portfolio. Its application to a portfolio of European corporate loans suggests that IFRS 9 will tend to frontload the impact of credit losses on P/L and CET1 right at the beginning of deteriorating phases of the economic cycle, which raises concerns about its procyclicality. Such impact, however, seems absorbable for banks with fully loaded capital conservation buffers.

Keywords: IFRS 9, credit loss allowances, expected credit losses, incurred losses, rating migrations, procyclicality.

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1 Introduction

This paper develops a recursive model for the assessment of the implications of the new measurement of credit impairment losses established by IFRS 9, the new international standard for the valuation of financial assets and liabilities, which will come into force in January 2018.[1] The key innovation of IFRS 9 is the shift from an incurred loss approach to an expected loss approach. Under IFRS 9 impairment allowances will be computed using a method involving two projection horizons. For exposures that have not suffered a significant increase in credit risk, impairment allowances will equal the one-year expected losses discounted at the effective contractual interest rate. For exposures that have suffered a significant deterioration of credit quality, impairment allowances will equal the life-time expected losses, also discounted at the effective contractual interest rate.

The recursive model described in this paper contains the minimal ingredients needed to assess impairment allowances under the above and alternative methods in a context where the differences between them have implications for both the average levels and the dynamics of the allowances associated with a given loan portfolio. The model is calibrated to analyze the behavior of a typical portfolio of European loans over the business cycle so as to assess the potential implications of the new measurement of impairments on the dynamics of banks' profit or loss (P/L) and common equity Tier 1 (CET1).

A difficulty for modeling the measurement method proposed by IFRS 9 is the complexity coming from having to keep track not only of the credit quality of a given loan but also of its credit quality at origination and its effective contractual interest rate. This difficulty introduces high dimensionality in the state space required to describe compactly the evolution over time of a loan portfolio. In general, a cohort of loans of a given rating, even if assumed to be composed of ex ante identical loans with the same effective contractual rate and to have a credit quality that evolves according to a cohort-independent ratings-migration matrix, would have to be distinguished from a cohort of loans originated with different effective contractual rates, even if their origination rating were the same.

Ideally, one would like to characterize the performance of alternative credit allowance

[1] See IASB (2014).

measurement methods in a setup where the pricing of the loans and the dynamics of the composition of the portfolio of loans of a representative holder (say, a bank) could be endogenously established in a way consistent with the background assumptions regarding the ratings-migration matrix, the loss-given-default parameters, and the maturity of the loans, as well as the evolution of aggregate risk and its impact on the previous parameters. Ideally, one would like to be explicit about the newly originated loans that enter the portfolio, possibly replacing the loans that mature or are resolved.

In a stationary situation without aggregate risk, one would like to be able to obtain the ergodic distribution of loans over the categories relevant for the measurement of their credit loss allowances under IFRS 9 and alternative methods. One would also like to be able to characterize the dynamic response of the system to shocks that either perturb punctually the composition of the loan portfolio (like the unanticipated once-and-for-all shocks commonly analyzed in macroeconomic theory) or affect more recurrently, in the form of systematic aggregate risk, the dynamics of the system. Besides, one would like to keep the model just rich enough to be suitable for calibration, i.e. for providing a tentative quantitative (and not only qualitative) assessment of the implications of IFRS 9 versus other methods for the measurement of credit loss allowances.

We achieve all this using a simple recursive ratings-migration model which is highly tractable thanks to a rather compact description of possible credit risk categories and, in the version with aggregate risk, a stylized description of the economic cycle as a two-state Markov chain.[2] A largely simplifying shortcut is the modeling of loan maturity as random (as in Leland and Toft, 1996), which prevents us from having to keep track of loan vintages.[3]

We calibrate the versions of the model without and with aggregate risk so as to match the characteristics of a typical portfolio of corporate loans of European banks. In the version with aggregate risk, we use evidence on the sensitivity of migration matrices and credit loss parameters to business cycles as in Bangia et al. (2002). The results point to relevant differences between IFRS 9 and alternative measurement methods (incurred loss, one-year

[2]See Trueck and Rachev (2009) as a general reference, and Gruenberger (2015) for an early application to the analysis of IFRS 9.

[3]Instead, in the version with aggregate risk, we need to keep track of the aggregate state of the economic cycle in which the loans are originated, since this affects the interest rate relevant for the discounting of their expected credit losses.

expected loss, and lifetime expected loss) regarding the level of the allowances and their dynamic responses to shocks.[4] More forward-looking methods, such as IFRS 9 (or the lifetime expected loss method envisaged by FASB for the US) imply significantly larger impairment allowances and sharper on-impact responses to negative shocks to (expected) credit quality, including those associated with changes in the aggregate state of the economy.

Under the current calibration of the model with aggregate risk, the arrival of a typical recession implies an on-impact increase in IFRS 9 impairment allowances whose unfiltered effect on CET1 would be equivalent to about a third of a bank's fully loaded capital conservation buffer. This means that the impact is sizeable but also suitably absorbable if such buffer is available when the shock hits. As we show, the arrival of contractions with anticipated severity or duration exceeding the average will tend to produce sharper responses, while having earlier notice of the arrival of a future contraction will tend to smooth out its impact.

These results suggest that, if regulatory filters do not offset or smooth away the cyclical impact of impairment allowances on CET1, IFRS 9 may lead banks to experience more sudden falls in regulatory capital right at the end of expansionary phases of the credit or business cycles. Banks can, of course, try to prepare for this by holding higher precautionary capital buffers in good times. Alternatively, they may adjust, when the time comes, by cutting on dividends or by issuing new equity, although there is ample anecdotal evidence and some formal empirical evidence on the fact that, when confronted with these choices, they undertake at least part of the adjustment by reducing their risk-weighted assets (e.g., cutting on the origination of new loans or rebalancing it towards safer ones).[5] In this case, very much through the same type of mechanisms extensively discussed in the literature on the procyclical effects of risk-sensitive bank capital requirements and the countercyclical effects of dynamic provisions, IFRS 9 might imply negative feedback effects on the supply

[4] Each of the alternative methods can be associated to existing or forthcoming accounting practices. Incurred loss was the standard under the current IAS 39, US GAAP, and most other national GAAPs. One-year expected loss is the method behind the internal-ratings based approach to capital requirements. Lifetime expected losses is the method envisaged by FASB to replace the incurred loss method in the US.

[5] See, for example, Mésonnier and Monks (2015), Gropp et al. (2016), and the references therein. The evidence in the second of these papers is consistent with average bank responses to the ESRB Questionnaire on Assessing Second Round Effects that accompanied the EBA stress test in 2016 regarding the way in which banks would expect to reestablish their desired levels of capitalization after exiting the adverse scenario.

of credit right when the cycle starts deteriorating.[6] Therefore it cannot be ruled out that, opposite to its motivational goals, IFRS 9 contributes to amplify rather than to dampen the cyclicality of credit supply.

In any case, from a normative perspective, this potential shortcoming of the new approach would have to be compared with the gains from provisioning future credit losses earlier and more cautiously, including those of having financial statements that more timely and reliably reflect the weakness or strength of the reporting institutions.[7] Offering such comprehensive evaluation exceeds the scope of this paper. Thus, the results in this paper should not be interpreted as a comprehensive assessment of the benefits and costs of IFRS 9 but as a first quantitative analysis of its potential procyclical effects. This analysis can be useful in the context of discussions on the adaptations that microprudential regulation or macroprudential policies may need under the new accounting standards (see, for example, BCBS, 2016).

The paper is organized as follows. Section 2 describes the baseline model without aggregate risk. Section 3 develops the formulas for the measurement of impairment losses under the various approaches that we compare and for assessing their effects on P/L and CET1. Section 4 explores the effect of an ad hoc shock to the credit quality of bank loans in the calibrated version of the baseline model. Section 5 presents and calibrates the model with aggregate risk and uses it to analyze the response to the arrival of a typical recession under the various measures. After having looked at banks operating under the internal-ratings based approach to capital requirements as a benchmark, Section 6 analyzes the results for the case of a bank operating under the standardized approach. Section 7 describes several extensions. Section 8 discusses the macroprudential implications of the results. Section 9 concludes the paper.

[6] Contributions to the literature on the procyclical effects of capital requirements include Kashyap and Stein (2004) and Repullo and Suarez (2013). Jiménez et al (2017) document the countercyclicality associated with the Spanish statistical provisions, with results suggesting that the effects of changes in capital pressure on credit are significantly more pronounced in recessions that in expansions.

[7] See Laeven and Majnoni (2001) and Huizinga and Laeven (2012) for evidence on bank provisioning practices and a discussion of their implications. See BCBS (2015) for a literature review.

2 Baseline model without aggregate risk

This section develops a simple recursive model of a bank's loan portfolio. In later sections, we will derive formulas and other ingredients necessary for measuring credit impairments under the various methods that we aim to compare and for assessing their impact on P/L and CET1. The model rests on ten assumptions that fully describe the elements relevant to understand the dynamics of loan origination, ratings migration, default, maturity, and pricing at origination of the loans that make up the loan portfolio. The tree in Figure 1 summarizes the contingencies relevant in the life of a loan (variables on each branch describe the relevant marginal conditional probabilities).

Model assumptions:

1. In each date t, existing loans belong to one of three credit rating categories: standard (j=1), sub-standard (j=2) or non-performing (j=3). We denote the measure of loans that belong to each category as x_{jt}.

2. In each date t, the bank originates a continuum of standard loans of measure $e_{1t} > 0$, with a principal normalized to one and a constant interest payment per period equal to c. So, in the language of IFRS 9, c is the effective contractual interest rate at which future expected losses will be discounted. In the analysis of steady states, we will assume a steady flow of entry of new loans $e_{1t} = e_1$ at each t.

3. Each loan's exposure at default (EAD) is constant and equal to one up to maturity.

4. Loans mature randomly and independently. Specifically, loans rated j=1, 2 mature at the end of each period with a constant probability δ_j.[8] This implies that conditional on remaining in rating j, a loan's expected life span is of $1/\delta_j$ periods. By the law of large numbers, the fraction of loans of a given rating j that mature at the end of each period is δ_j. In steady state, this produces a stream of maturity cash flows very similar to those that would emerge with a portfolio of perfectly-staggered loans with identical deterministic maturities at origination.

[8] Allowing for $\delta_1 \neq \delta_2$ may help capture the possibility that longer maturity loans get early redeemed with different probabilities depending on their credit quality.

5. In the case of non-performing loans (j=3), δ_3 represents the independent per period probability of a loan being resolved, in which case it pays back a fraction $1 - \lambda$ of its principal and exits the portfolio. So the constant λ is the loss rate at resolution, which in the baseline model coincides with the loan's expected loss given default (LGD).

6. Each loan rated j=1, 2 at t that matures at t+1 defaults independently with probability PD_j. Maturing loans that do not default pay back their principal of one plus interest c. Each defaulted loan is resolved within the same period with an independent probability $\delta_3/2$.[9] Otherwise, it enters the stock of non-performing loans (j=3).

7. Each loan rated j=1, 2 at t that does not mature at $t + 1$ goes through one of the following exhaustive possibilities:

   (a) Default, which occurs independently with probability PD_j. As when a maturing loan defaults, a non-maturing loan that defaults is resolved within the same period with probability $\delta_3/2$, yielding $1 - \lambda$. Otherwise, it enters the stock of non-performing loans (j=3).

   (b) Migration to rating $i \neq j$ (i=1,2), which occurs independently with probability a_{ij}. In this case the loan pays interest c and continues for one more period with its new rating.

   (c) Staying in rating j, which occurs independently with probability $a_{jj} = 1 - a_{ij} - PD_j$. In this case the loan pays interest c and continues for one more period with its previous rating.

8. Non-performing loans (j=3) pay no interest and never return to the performing categories. They accumulate in category j=3 up to their resolution.[10]

9. The contractual interest rate c is established at origination as in a perfectly competitive environment with risk-neutral banks that face an opportunity cost of funds between

[9] We divide δ_3 by two to reflect the fact that if loans default uniformly during the period between t and t+1, they will have on average just half a period to be resolved. The model can trivially accommodate alternative assumptions on same-period resolutions.

[10] For calibration purposes, one might account for potential gains from the unmodeled interest accrued while in default or from returning to performing categories by adjusting the loss rate λ.

any two periods equal to a constant r. The originating bank is assumed to hold the loans up to their maturity, hence satisfying the "business model" condition required by IFRS 9 for the valuation of basic lending assets at amortized cost.

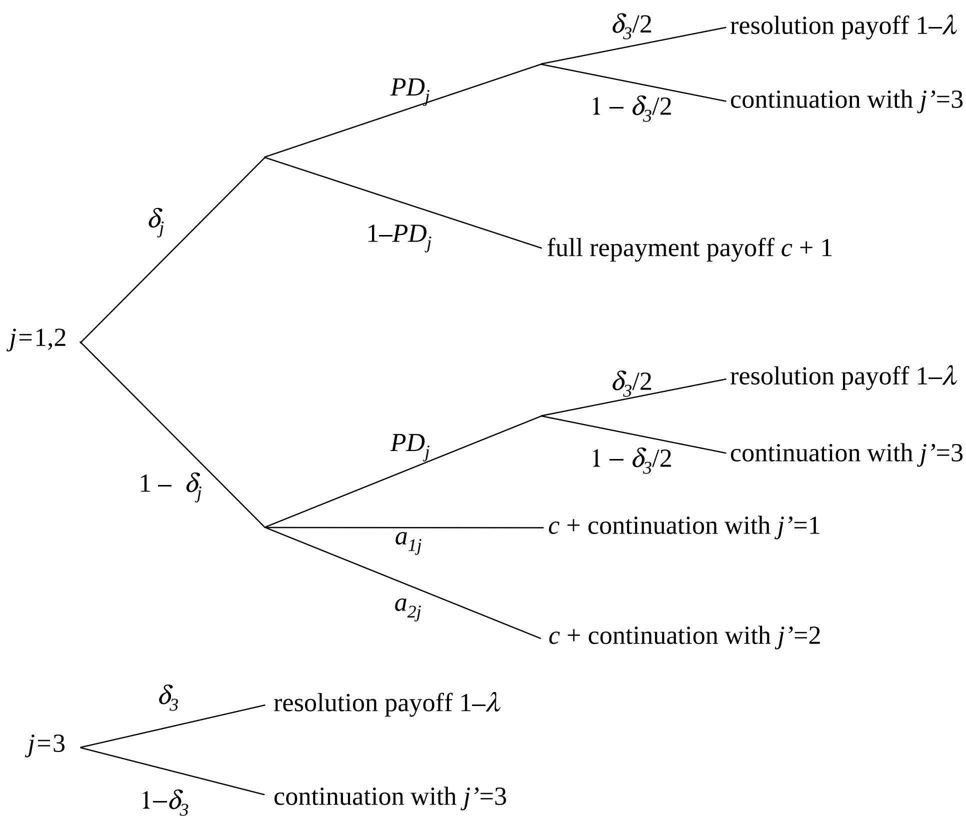

Figure 1. Possible transitions of a loan rated j
Possible contingencies between two dates and their implications for payoffs and continuation value.

10. Finally, one period corresponds to a calendar year, and dates t, $t+1$, $t+2$, etc. denote end-year accounting reporting dates (so "period t" ends at "date t").

In the version of the model with aggregate risk that we present in Section 5, we will allow all the parameters in the tree depicted in Figure 1 to vary with the aggregate state of the economy.

2.1 Portfolio dynamics without aggregate risk

The model presented so far has no aggregate risk. By the law of large numbers, the evolution of the loans belonging to each rating can be represented by the difference equation:

$$x_t = Mx_{t-1} + e_t \tag{1}$$

where

$$x_t = \begin{pmatrix} x_{1t} \\ x_{2t} \\ x_{3t} \end{pmatrix} \tag{2}$$

is the vector that describes the loans in each rating category j=1,2,3,

$$M = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ m_{21} & m_{22} & m_{23} \\ m_{31} & m_{32} & m_{33} \end{pmatrix} = \begin{pmatrix} (1-\delta_1)a_{11} & (1-\delta_2)a_{12} & 0 \\ (1-\delta_1)a_{21} & (1-\delta_2)a_{22} & 0 \\ (1-\delta_3/2)PD_1 & (1-\delta_3/2)PD_2 & (1-\delta_3) \end{pmatrix} \tag{3}$$

accounts for the migrations across categories of the non-matured, non-defaulted loans, and

$$e_t = \begin{pmatrix} e_{1t} \\ 0 \\ 0 \end{pmatrix} \tag{4}$$

accounts for the new loans originated at each date, which we write reflecting that at origination all loans have rating j=1.

2.2 Steady state portfolio without aggregate risk

If the amount of newly originated loans is equal at all dates ($e_t = e$ for all t), the loan portfolio will asymptotically converge to a time-invariant or steady-state portfolio x^* that can be obtained as the vector that solves:

$$x = Mx + e \Leftrightarrow (I - M)x = e, \tag{5}$$

that is,

$$x^* = (I - M)^{-1}e. \tag{6}$$

3 Measuring impairment losses

In this section we derive formulas for the measurement of the impairments generated by the previously described loan portfolio under different approaches. We also discuss how to endogenously determine a contractual loan rate c consistent with our assumptions on the competitive pricing of loans at origination. Finally, we provide some formulae relevant to assess the impact of impairment measurement on the bank's P/L and CET1.

3.1 Incurred losses

The incurred loss approach is the one that has characterized accounting standards in most jurisdictions in recent years. By January 2018 IFRS 9 is scheduled to replace it in all jurisdictions that have IAS as an accounting standards setter.

Under the narrowest interpretation, allowances measured on an incurred loss basis are restricted to non-performing loans (NPLs). In our setup, the incurred losses reported at t are

$$IL_t = \lambda x_{3t}, \tag{7}$$

since the loss rate λ is the expected LGD of the bank's NPLs at date t. Notice that under our assumptions the losses associated with loans defaulted between dates $t-1$ and t that are resolved within such period, $\lambda(\delta_3/2)(PD_1 x_{1t-1} + PD_2 x_{2t-1})$, do not enter x_{3t} and, hence, will be directly annotated in the P/L of year t.

3.2 Discounted one-year expected losses

The one-year expected-ahead approach is, roughly speaking, the one prescribed for regulatory purposes for banks following the internal ratings-based (IRB) approach to capital requirements.[11]

For loans performing at t, impairment allowances are measured on a discounted one-year expected basis. Thus, they are forward looking, but with the forecasting horizon limited to one year. For loans with j=1, 2 the allowance is computed taking into account the losses due to default events expected to occur within the immediately incoming year. For non-performing loans (j=3), the allowance equals the whole (non-discounted) loss given default

[11] In fact, BCBS prescriptions on regulatory provisions establish that the PDs that must feed the above formula must be through-the-cycle (rather than point-in-time) estimates of the corresponding probability of default. By the same logic, they establish that the LGDs must conservatively reflect a distressed liquidation scenario rather than a central scenario. Prescriptions for discounting are also slightly different. To simplify the analysis, we abstract from all these differences.

of the loans, so we can write

$$EL_t^{1Y} = \lambda \left[\beta(PD_1 x_{1t} + PD_2 x_{2t}) + x_{3t}\right] \tag{8}$$

where $\beta = 1/(1+c)$ is the discount factor based on the contractual interest rate c. In Section 3.5 we derive an expression for the endogenous value of c consistent with our assumptions on loan pricing.

In matrix notation, which will be useful when extending the loss forecasting horizon to several years, the above credit loss allowances can also be expressed as

$$EL_t^{1Y} = \lambda \left(\beta b x_t + x_{3t}\right), \tag{9}$$

where

$$b = (PD_1, PD_2, 0). \tag{10}$$

3.3 Discounted lifetime expected losses

The lifetime expected loss approach is the one that the US accounting standards setter, FASB, has planned for the replacement of the current US GAAP incurred loss approach.

For loans performing at t, credit loss allowances under the lifetime expected loss approach are the sum of the discounted expected losses that the loans are projected to cause in each of the years in the future. Instead, for non-performing loans, the allowance covers the whole (non-discounted) loss given default of the affected loans. So the allowances can be found as:[12]

$$EL_t^{LT} = \lambda b \left(\beta x_t + \beta^2 M x_t + \beta^3 M^2 x_t + \beta^4 M^3 x_t + ...\right) + \lambda x_{3t}, \tag{11}$$

which reflects that the losses that currently performing loans are expected to cause at any future date $t + \tau$ can be found as $\lambda b M^{\tau-1} x_t$, where b contains the relevant one-year-ahead PDs (see (10)) and $M^{\tau-1} x_t$ gives the projected composition of the portfolio at each future date $t + \tau - 1$.

In this recursive setup, the lifetime expected losses in the first term of (11) can be expressed as

$$EL_t^{LT} = \beta \lambda b(I + \beta M + \beta^2 M^2 + \beta^3 M^3 + ...)x_t + \lambda x_{3t}, \tag{12}$$

[12] In the FASB proposal, the discount factor β is not based on the effective contractual interest rate of the loan, but on a reference risk-free rate. However, we will abstract from this feature and use a common definition of β throughout all the impairment measures compared in this paper.

where the parenthesis contains the infinite sum of a geometric series of matrices, which can be found as

$$B = (I - \beta M)^{-1}. \tag{13}$$

Thus, we can compute EL_t^{LT} as

$$EL_t^{LT} = \lambda \left(\beta b B x_t + x_{3t} \right). \tag{14}$$

Obviously, $B \geq I$, so $EL_t^{LT} \geq EL_t^{1Y}$.

3.4 Discounted expected losses under IFRS 9

As already mentioned, IFRS 9 adopts, for performing loans, a mixed-horizon approach that combines the one-year-ahead and life-time approaches described above. Specifically, it applies the one-year-ahead measurement to loans that have not suffered a significant increase in credit risk since origination ("stage 1" loans), which for us are the standard loans x_{1t}. It applies the life-time measurement to performing loans with deteriorated credit quality ("stage 2" loans), which for us are the sub-standard loans x_{2t}. Finally, for non-performing loans ("stage 3" loans), x_{3t}, the allowance simply equals the whole (non-discounted) expected LGD, as under any of the other approaches.

Combining the formulas obtained in (9) and (14), the impairment allowances under IFRS 9 can be described as

$$EL_t^{IFRS9} = \lambda \beta b \begin{pmatrix} x_{1t} \\ 0 \\ 0 \end{pmatrix} + \lambda \beta b B \begin{pmatrix} 0 \\ x_{2t} \\ 0 \end{pmatrix} + \lambda x_{3t}, \tag{15}$$

which, together with $EL_t^{LT} \geq EL_t^{1Y}$, implies $EL_t^{1Y} \leq EL_t^{IFRS9} \leq EL_t^{LT}$.

3.5 Loan pricing

Taking advantage of the recursivity of the model, we can obtain the bank's ex-coupon value of loans rated j at any given date, v_j, by solving the following system of Bellman-type equations:

$$v_j = \mu \left[(1 - PD_j)c + (1 - PD_j)\delta_j + PD_j(\delta_3/2)(1 - \lambda) + m_{1j}v_1 + m_{2j}v_2 + m_{3j}v_3 \right], \tag{16}$$

for j=1, 2, and

$$v_3 = \mu \left[\delta_3(1-\lambda) + (1-\delta_3)v_3 \right], \tag{17}$$

where $\mu = 1/(1+r)$ is the discount factor of the risk neutral bank and the square brackets in (16) and (17) contain the continuation payoffs or value obtained under the contingencies that, in each case, can occur one period ahead (weighted by the corresponding probabilities).[13]

The first term within the square brackets in (16) accounts for the interest that the loan currently rated j will pay in the next date if it continues performing. The second term captures the terminal value obtained if the loan matures without defaulting. The third term accounts for the terminal value recovered if the loan defaults and gets resolved within the period. The fourth and fifth terms reflect the continuation value obtained if the loan remains non-matured and gets (or retains) rating 1 and 2, respectively, for the next period. The last term measures the continuation value obtained if the loan defaults but it is not resolved within the period, becoming a non-performing loan.

The first term within the square brackets in (17) accounts for the terminal value recovered if a non-performing loan is resolved within the period. The last term reflects the continuation value of the non-performing loan if it remains unresolved at $t+1$.

Perfect competition implies that the value of extending a loan of size one rated as standard (j=1) at origination must equal the value of its principal (one), so that the bank obtains zero net present value from its origination. Solving for c delivers the endogenous contractual interest rate that enters the discount factor $\beta = 1/(1+c)$ used in the various expectation-based impairment measures established above.

3.6 Implications for P/L and CET1

To explore the implications of impairment measurement for the dynamics of the P/L account and for CET1, we need to make further assumptions regarding the bank that holds the loan portfolio discussed so far and its capital structure. To simplify the discussion, we abstract from bank failure and assume that the bank's only assets at the end of any period t are the loans described by vector x_t and that its liabilities are made exclusively of (i)

[13] For calibration purposes, the discount rate r does not need to equal the risk-free rate. One might adjust the value of r to reflect the marginal weighted average costs of funds of the bank or even an extra element capturing (in reduced form) a mark-up applied on that cost if the bank is not perfectly competitive.

(risk-free) one-period debt, d_t, that promises to pay interest r per period, (ii) impairment allowances a_t computed under one of the measurement approaches described above (so $a_t = IL_t, EL_t^{1Y}, EL_t^{LT}, EL_t^{IFRS9}$), and (iii) CET1, k_t. So the bank's balance sheet at the end of any period t can be described as

$$\begin{array}{c|c} x_{1t} & d_t \\ x_{2t} & a_t \\ x_{3t} & k_t \end{array} \tag{18}$$

with the law of motion of x_t described by (1) and the law of motion of k_t given by

$$k_t = k_{t-1} + PL_t - \text{div}_t + \text{recap}_t, \tag{19}$$

where PL_t is the result of the P/L account at the end of period t, $\text{div}_t \geq 0$ are cash dividends paid at the end of period t, and $\text{recap}_t \geq 0$ are injections of CET1 at the end of period t. Under these assumptions, the dynamics of d_t can be recovered residually from the balance sheet identity, $d_t = \Sigma_{j=1,2,3} x_{jt} - a_t - k_t$.

The result of the P/L account can in turn be written as

$$PL_t = \left\{ \sum_{j=1,2} \left[c(1-PD_j) - \frac{\delta_3}{2} PD_j \lambda \right] x_{jt-1} - \delta_3 \lambda x_{3t-1} \right\} - r \left(\sum_{j=1,2,3} x_{jt-1} - a_{t-1} - k_{t-1} \right) - \Delta a_t, \tag{20}$$

where the first term contains the income from performing loans net of realized losses on defaulted loans resolved during period t, the second term is the interest paid on d_{t-1}, and the third term is the variation in credit loss allowances between periods $t-1$ and t.

To model dividends, div_t, and equity injections, recap_t, in a simple manner, we assume that the bank manages the evolution of its CET1 using a simple sS-rule entirely determined by existing capital regulations.[14] Specifically, current Basel III prescriptions include minimum capital requirements and the so-called capital conservation buffer (CCB). Minimum capital requirements force the bank to operate with a CET1 of at least $\underline{k}_t$, while the CCB requires the bank to retain profits, whenever feasible, until reaching a fully loaded buffer

[14] This rule can be rationalized as the one that minimizes the equity capital committed to support the loan portfolio. Its working here is consistent with the absence of fixed costs associated with the rasing of new equity. If such costs were introduced, the optimal rule would imply, like in Fischer, Heinkel, and Zechner (1989), discrete recapitalizations to a level in the interior of the two bands when the lower band were otherwise passed.

equal to 2.5% of the bank's risk-weighted assets (RWAs). This means that a bank with positive profits must accumulate them until its CET1 reaches a level $\overline{k}_t = 1.3125\underline{k}_t$.[15]

Thus, we assume the bank's dividends and equity injections to be determined as

$$\text{div}_t = \max[(k_{t-1} + PL_t) - 1.3125\underline{k}_t, 0] \tag{21}$$

and

$$\text{recap}_t = \max[\underline{k}_t - (k_{t-1} + PL_t), 0], \tag{22}$$

respectively.

Minimum capital requirement under the IRB approach For banks or portfolios operated under the IRB approach, the IRB formula specified in BIS (2004, paragraph 272) establishes that the regulatory capital requirement must be

$$\underline{k}_t^{IRB} = \sum_{j=1,2} \gamma_j x_{jt}, \tag{23}$$

with

$$\gamma_j = \lambda \frac{1 + [(1/\delta_j) - 2.5]m_j}{1 - 1.5m_j} \left[\Phi \left(\frac{\Phi^{-1}(PD_j) + \text{cor}_j^{0.5}\Phi^{-1}(0.999)}{(1 - \text{cor}_j)^{0.5}} \right) - PD_j \right], \tag{24}$$

where $m_j = [0.11852 - 0.05478 \ln(PD_j)]^2$ is a maturity adjustment coefficient, $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distribution, and cor_j is a correlation coefficient fixed as $\text{cor}_j = 0.24 - 0.12(1 - \exp(-50PD_j))/(1 - \exp(-50))$.

Minimum capital requirement under the standardized (SA) approach For banks or portfolios operated under the SA approach, the regulatory minimum capital requirement applicable to loans to corporations without an external rating is just 8% of the exposure net of its "specific provisions" (a regulatory concept related to impairment allowances). Assuming all the loans in x_t correspond to unrated borrowers and that all the impairment allowances qualify as specific provisions, this implies

$$\underline{k}_t^{SA} = 0.08 \left(\sum_{j=1,2,3} x_{jt} - a_t \right). \tag{25}$$

[15] Under Basel III, RWAs equal 12.5 (or 1/0.08) times the bank's minimal required capital $\underline{k}_t$. Thus a fully loaded CCB amounts to a multiple $0.025 \times 12.5 = 0.3125$ of $\underline{k}_t$.

Formulas (23) and (25) will allow us to assess the impact of different impairment measurement methods on the dynamics of PL_t, k_t, div_t, and recap_t under each of the approaches to capital requirements.

It is important to notice that, as a first approximation, our analysis abstracts from the existence of "regulatory filters" dealing with the implications of possible discrepancies between "accounting" and "regulatory" provisions and their effects on "regulatory capital." In this sense, our assessment below can be seen as an evaluation of the impact of accounting rules on bank capital dynamics in the polar case in which the bank capital regulators accept the new accounting provisions (and the resulting accounting capital) as the provisions (and available capital) to be used for regulatory purposes as well.[16]

4 A first quantitative exploration

The model described so far features a relatively small number of parameters. Table 1 describes their value under a parameterization intended to represent a typical portfolio of corporate exposures of EU banks. Given the absence of detailed publicly available microeconomic information on such a portfolio, the calibration relies on aiming to match aggregate variables taken from recent EBA reports and ECB statistics using ratings migration and default probabilities consistent with the Global Corporate Default reports produced by Standard & Poor's (S&P) over the period 1981-2015.[17]

Banks' discount rate r is fixed at 1.8% so as to obtain a contractual loan rate c equal to 2.54%, which is very close to the 2.52% average interest rate of new corporate loans made by Euro Area banks in the period from January 2010 to September 2016.[18] The probabilities of

[16] In the case of IRB banks, the current regulatory regime (which might be revised to accommodate IFRS 9) establishes that regulatory provisions are one-year expected losses, EL_t^{1Y}. If EL_t^{1Y} exceeds the accounting allowances, a_t, the difference, $EL_t^{1Y} - a_t$, must be subtracted from CET1. In contrast, if $EL_t^{1Y} - a_t < 0$, the difference can be added back as Tier 2 capital up to a maximum of 0.6% of the bank's credit RWAs. In the case of SA banks, there is a filter for the so-called general provisions (that in our analysis we assume, for simplicity, to be zero), which can be added back as Tier 2 capital up to a maximum of 1.25% of credit RWAs.

[17] We use reports equivalent to S&P (2016) published in years 2003 and 2005-2016.

[18] We use the Euro area (changing composition), Annualised agreed rate (AAR) / Narrowly defined effective rate (NDER) on Euro denominated loans other than revolving loans and overdrafts, convenience and extended credit card debt, made by banks to non-financial corporations (http://sdw.ecb.europa.eu/quickview.do?SERIES_KEY=124.MIR.M.U2.B.A2A.A.R.A.2240.EUR.N)

default and yearly probabilities of migration across our standard and substandard categories are extracted from S&P ratings-migration data using the procedure that we describe in detail in Appendix A. They are consistent with assimilating our standard category (j=1) to ratings AAA to BB in the S&P classification and our substandard category (j=2) to ratings B to C.

Table 1
Calibration of the model without aggregate risk

Banks' discount rate	r	1.8%
Yearly probability of migration 1 → 2 if not maturing	a_{21}	7.37%
Yearly probability of migration 2 → 1 if not maturing	a_{12}	6.29%
Yearly probability of default if rated j=1	PD_1	0.85%
Yearly probability of default if rated j=2	PD_2	7.29%
Loss given default	λ	36%
Average time to maturity if rated j=1	$1/\delta_1$	5 years
Average time to maturity if rated j=2	$1/\delta_2$	5 years
Yearly probability of resolution of NPLs	δ_3	44.6%
Newly originated loans per period (all rated j=1)	e_1	1

In a nutshell, we reduce the 7 × 7 ratings-migration probabilities and the seven probabilities of default in S&P data to the 2 × 2 migration probabilities and two probabilities of default that appear in matrix M (equation (3)) by computing weighted averages that take into account the steady state composition that the loan portfolio would have under its 7-ratings representation. To get such composition, we assume that loans have an average duration of 5 years (or δ_1=δ_2=0.2) as in Table 1, a rating BB at origination, and that then evolve (by improving or worsening their credit quality before defaulting or maturing) exactly as in our model but with the seven non-default rating categories present in the original S&P data.

Under these assumptions, we obtain average yearly PDs for our standard and substandard categories of 0.9% and 7.3%, respectively. As reported in Table 2, given the composition of the "reduced" steady-state portfolio, the average annual loan default rate equals 1.9%, which is below the average 2.5% PD on non-defaulted corporate exposures that EBA (2013, Figure 12) reports for the period 2009h1-2012h2 for a sample of EU banks operating under the IRB approach.

Table 2
Endogenous variables under the no-aggregate-risk calibration
(IRB bank, all variables in %)

Yearly contractual loan rate, c	2.54
Steady-state portfolio shares (% of total loans)	
Standard loans, $x_1^*/(\Sigma_{j=1,2,3}x_j^*)$	81.29
Sub-standard loans, $x_2^*/(\Sigma_{j=1,2,3}x_j^*)$	15.53
Non-performing loans, $x_3^*/(\Sigma_{j=1,2,3}x_j^*)$	3.18
Average yearly PD on non-defaulted loans, $(\Sigma_{j=1,2}PD_jx_j^*)/(\Sigma_{j=1,2}x_j^*)$	1.88
Average yearly PD on total loans, $(\Sigma_{j=1,2}PD_jx_j^* + x_3^*)/(\Sigma_{j=1,2,3}x_j^*)$	5.00
Steady-state allowances (% of total loans):	
Incurred losses	1.14
One-year expected losses	1.78
Lifetime expected losses	4.64
IFRS 9 allowances	2.67
Stage 1 allowances	0.24
Stage 2 allowances	1.28
Stage 3 allowances	1.14
IRB capital requirement for standard loans, γ_1	7.57
IRB capital requirement for sub-standard loans, γ_2	12.86
IRB minimum capital requirement (% of total loans), $\underline{k}$	8.15
IRB minimum capital requirement + CCB (% of total loans), $\overline{k}$	10.70

The LGD parameter λ is set equal to 36%, which roughly matches the average LGD on corporate exposures that EBA (2013, Figures 11 and 13) reports for 2009h1-2012h2 for the same sample mentioned above. Finally, we set δ_3 equal to 44.6% so as to produce a steady state fraction of non-performing loans (NPLs) consistent with the 5% average probability of default including defaulted exposures that EBA (2013, Figure 10) reports for the earliest period in its study, 2008h2.[19] This value of δ_3 implies an average time to resolution for NPLs of 2.24 years, which is very close to the 2.42 years estimated average duration of corporate insolvency proceedings across EU countries documented by EBA (2016, Figure 13).

Finally, the assumed flow of newly originated loans, e_1=1, provides just a normalization, solely affecting the size of the steady state loan portfolio.

[19] We take this observation, right before experiencing the full negative impact of the Global Financial Crisis, as the best proxy in the data for the model's "steady state." As shown in Table 2, with this procedure, we obtain a 3.2% share of defaulted exposures in the steady state portfolio, right inbetween the 2.5% and 4.4% reported by EBA (2013, Figure 8) for corporate loans in 2008h2 and 2009h1, respectively.

The second block of Table 2 reports the size of the credit impairment allowances in steady state under each of the measurement methods that we compare. The third block reports the IRB capital requirements, the implied overall minimum capital requirement, $\underline{k}$, and the minimum requirement plus CCB, $\overline{k}$, that we use to model the dynamics of CET1.[20] The various impairment measures are clearly ranked, with sizeable differences between them. The steady state level of EL_t^{IFRS9} is closer to that of EL_t^{1Y} than to that of EL_t^{LT} because the steady state portfolio contains a not very large (15.5%) fraction of substandard loans ("stage 2" loans under IFRS 9).

As a first look into the implications of the model for the response of the various credit impairment measures to shocks that erode the expected credit quality of the loan portfolio, we consider the following thought experiment. Suppose the loan portfolio is at its steady state composition at some initial date t=–1. Suppose further that at t=0 the system is hit by a large unexpected once-and-for-all shock that makes an extra 35% of the standard-quality loans of the previous date to become substandard (instead of remaining standard one more period), so that their rating migrations typically driven by a_{11} and a_{21} become punctually driven by $a'_{11} = a_{11} - 0.35$ and $a'_{21} = a_{21} + 0.35$, respectively. Formally, this means perturbing m_{11} and m_{21} to $m'_{11} = (1 - \delta_1)(a_{11} - 0.35)$ and $m'_{21} = (1 - \delta_1)(a_{21} + 0.35)$ for just one period.

From t=1 onwards the system simply follows its own dynamics, according to the parameters described in Table 1, without further shocks. Notice, however, that the presence of an abnormally high amount of substandard loans will make the effects of the initial shock persistent over time. This can be seen in Panel A of Figure 2, which depicts the evolution of NPLs in this thought experiment.

The results regarding the evolution of the various impairment measures over the same time span appear in Panel B of Figure 2. Credit loss allowances IL_t, EL_t^{1Y}, EL_t^{LT}, and EL_t^{IFRS9} are reported as a percentage of the total initial loans. The levels of the series at t=–1 reflect the different sizes of the various measures in steady state.

[20] To keep the analysis focused, we first discuss the case of IRB banks, postponing to Section 6 the comparison with SA banks.

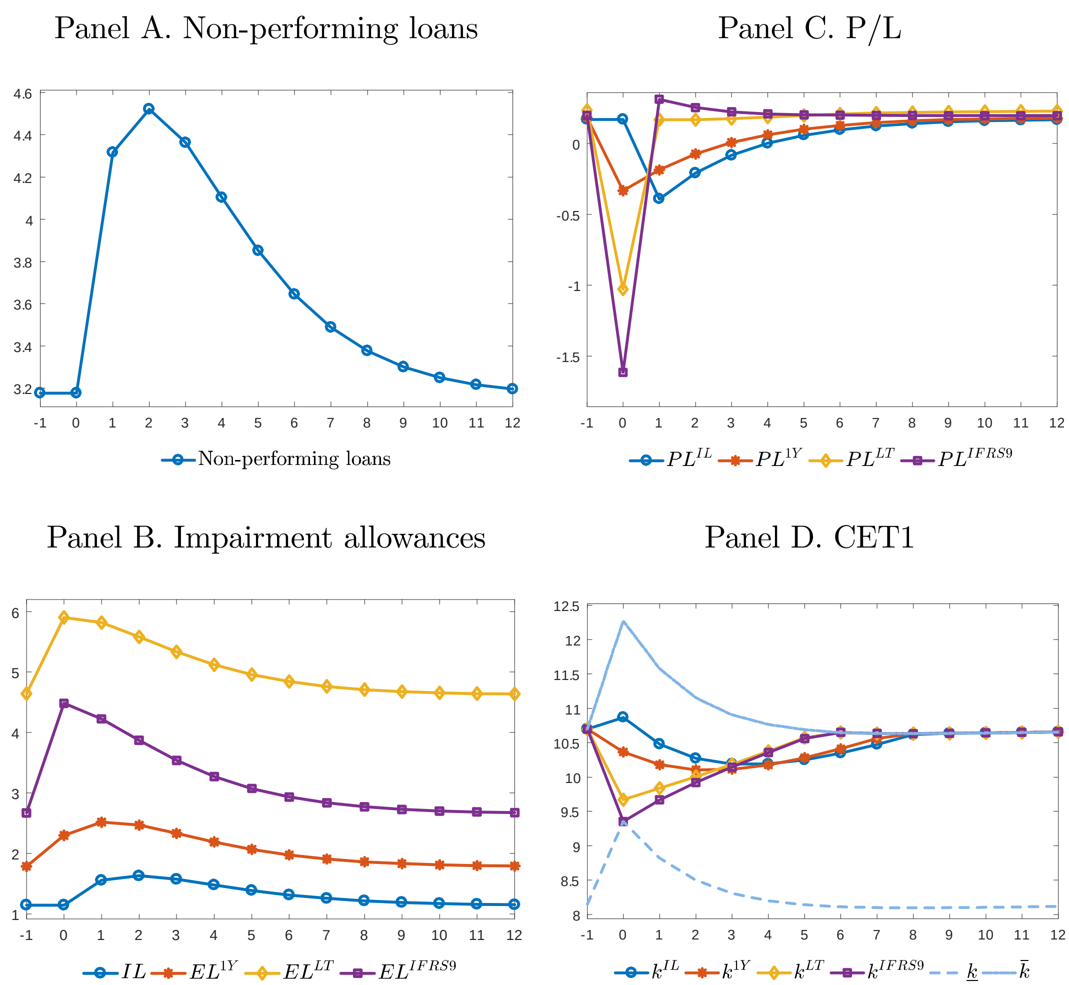

Figure 2. Effects of a negative shock to credit quality
Responses to an unexpected once-and-for-all shock to credit quality
(IRB bank, in % of initial exposures)

What the bottom panel of Figure 2 represents for t=0,1,2... is equivalent to a typical impulse response function in macroeconomic analysis. When the shock hits at t=0 all measures except IL_t, which reacts with one period delay given its backward looking nature, move upwards for one or two periods before entering a pattern of exponential decay, driven by maturity, defaults, migration of substandard loans back to the standard category, and the continued entry of new standard quality loans.[21]

[21] Variations of the experiment that simultaneously shut down or reduce new entry for a number periods can be easily performed without losing consistency. Experiencing lower loan origination after t=0 delays the process of reversion to the steady state but does not qualitatively affect the results.

The responses of EL_t^{1Y} and, when it comes, IL_t to the shock are much smaller than those of the further forward-looking measures. Interestingly, the on-impact response of EL_t^{IFRS9} (which increases by about 1.9 percentage points of initial exposures, pp) exceeds that of EL_t^{LT} (which increases by about 1.3pp). In contrast, EL_t^{1Y} increases by barely 0.5pp at its peak (at t=1) and IL_t increases by roughly 0.4pp at its peak (at t=2).

The implications of the various impairment measures for P/L are described in Panel C of Figure 2. Essentially each measure spreads over time the (same final) impact of the shock on P/L in a different manner. EL_t^{IFRS9} and, to a lower extent, EL_t^{LT} front load the impact of the shock to the extent of making P/L very negative on impact but then positive and even above normal for a number of periods afterwards. With EL_t^{1Y} (and IL_t), P/L gets a much smaller (and delayed) hit on impact but, in turn, remains negative for several periods. Interestingly, the measure allowing P/L to return to normal at a quickest speed in this experiment is EL_t^{LT}.

Panel D of Figure 2 shows the implications for CET1 for an IRB bank. Before the shock hits, at t=–1, the bank is assumed to have its CCB fully loaded, implying a buffer on top of the minimum required capital of more than 2.5% of total assets. The change in the bands $\underline{k}$ and $\overline{k}$ reflected in the figure are the result of the change in RWAs that follows the deterioration in the composition of the bank's loan portfolio. The differences in the effects of the alternative measures on CET1 are dramatic, essentially mirroring their impact on P/L.

In the case of IFRS 9, an abnormal extra shift of 35% of the loans from j=1 to j=2 at t=0 implies consuming the CCB in that very year and having to raise a (small) amount of new equity. Using the alterative measures, no equity issuance is required and the return to normal capital levels occurs solely via earnings retention.

Of course, the need for recapitalization or not under the various impairment measures in a thought experiment like this depends on the ad hoc size of the initial shock, so far fixed at 35% for purely illustrative reasons. However, the (weak) order of the recapitalization needs that each measurement method would imply happens to be invariant to the size of the shock. This can be seen in Figure 3, which shows the cumulative capital issuance needs implied by a shock like this under each measure (vertical axis) as a function of the additional fraction of standard loans that the shock converts into substandard (horizontal axis).

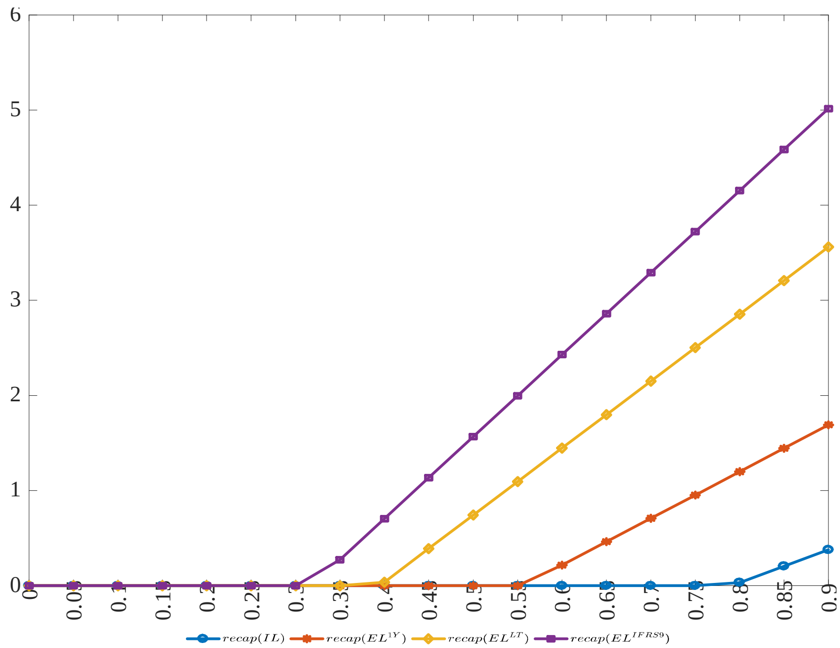

Figure 3. Recapitalization needs and the size of the shock
X-axis: fraction of standard loans abnormally turning substandard
Y-axis: sum of recapitalization needs
(IRB bank, in % of initial exposures)

5 Adding aggregate risk

The most natural way to incorporate aggregate risk in the model is by considering an aggregate state variable s_t whose evolution affects the key parameters governing portfolio dynamics and credit losses in the model. To keep things simple, we will assume that s_t follows a Markov chain with two states s=1,2 and time-invariant transition probabilities $p_{s's}$ =Prob$(s_{t+1} = s'|s_t = s)$. Under this representation, s=1 might, for example, refer to expansion or quiet periods, while s=2 refers to contraction or crisis periods.[22]

In Appendix B we extend the model and the formulae for the calculation of portfolio dynamics and impairment allowances to accommodate the case in which the parameters determining the (expected) maturity of the loans, their default probabilities, their migration across ratings, their probability of being resolved when in default, their loss rates upon

[22]See Bangia et al. (2002) for an empirical ratings migration model in which macroeconomic conditions are represented in this manner.

resolution, and the flow of entry of new loans between any dates t and $t+1$ may vary with the arrival state s_{t+1}.

An approach that allows us to keep the analysis recursive as in the baseline model is to expand the vectors describing loan portfolios so that components describe "loans originated in state z, currently in state s and rated j", for each possible (z, s, j) combination, instead of just "loans rated j". In parallel, we expand the transition matrices describing the dynamics of these portfolios to reflect the possible transitions of the aggregate state and their impact on all the relevant parameters. The need to keep track of the state at origination z comes from the need to discount the future credit losses of each loan using the effective contractual interest rate, which now varies with the aggregate state at origination and is denoted c_z.

5.1 Calibration with aggregate risk

Table 3 describes the calibration of the model with aggregate risk. As further explained in section A.3 of Appendix A, we allow for state-variation in the probabilities of loans migrating across rating categories and into default in a way consistent with the historical correlation between those variables (as observed in S&P ratings-migration data) and the US business cycle as dated by the National Bureau of Economic Research (NBER).[23] The dynamics of the aggregate state as parameterized in Table 3 imply average durations of expansion and contraction periods of 6.75 years and 2 years, respectively, meaning that the system spends about 77% of the time in state s=1. Expansions are characterized by significantly smaller PDs among both standard and substandard loans than contractions. Contractions almost double the probability of standard loans being downgraded (or, under IFRS 9, moved into stage 2) and reduce by about a third the probability of substandard loans recovering standard quality (or returning to stage 1).

To keep the potential sources of cyclical variation under control, we maintain as time invariant (and equal to their values in the calibration without aggregate risk) the parameters determining the effective maturity of performing loans, the speed of resolution of NPLs, the LGDs, and the flow of entry of new loans.

[23] See http://www.nber.org/cycles.html.

Table 3
Calibration of the model with aggregate risk

Parameters without variation with the aggregate state		
Banks' discount rate	r	1.8%
Persistence of the expansion state (s=1)	p_{11}	0.852
Persistence of the contraction state (s=2)	p_{22}	0.5

Parameters that may possibly vary with the aggregate state		If $s' = 1$	If $s' = 2$
Yearly probability of migration 1 → 2 if not maturing	a_{21}	6.16%	11.44%
Yearly probability of migration 2 → 1 if not maturing	a_{12}	6.82%	4.47%
Yearly probability of default if rated j=1	PD_1	0.54%	1.91%
Yearly probability of default if rated j=2	PD_2	6.05%	11.50%
Loss given default	λ	36%	36%
Average time to maturity if rated j=1	$1/\delta_1$	5 years	5 years
Average time to maturity if rated j=2	$1/\delta_2$	5 years	5 years
Yearly probability of resolution of NPLs	δ_3	44.6%	44.6%
Newly originated loans per period (all rated j=1)	e_1	1	1

5.2 Cyclicality of the various impairment measures

Table 4 reports unconditional means, standard deviations, and means conditional on each aggregate state, for a number of selected endogenous variables. The variation in the aggregate state causes a significant variation in the composition of the bank's loan portfolio. Not surprisingly, in the contraction state, stage 2 and stage 3 loans represent a larger share of the portfolio, and the realized overall default rate is more than twice as large as in the expansion state. As in prior sections, we so far focus the analysis of the implications for CET1 on the case of IRB banks, relegating the comparison with the case of SA banks to Section 6.

The mean relative sizes of the various impairment allowances are essentially the same obtained for the case without aggregate risk. Interestingly, impairments measured under IFRS 9 are the most volatile, followed closely by those measured under the lifetime expected approach. The least volatile measure is IL.

Table 4
Endogenous variables under the aggregate-risk calibration
(IRB bank, % of mean exposures unless indicated)

	Mean	St. Dev.	Conditional means	
			s=1	s=2
Yearly contractual loan rate c (%)			2.52	2.62
Share of standard loans (%)	81.35	3.48	82.68	76.85
Share of sub-standard loans (%)	15.46	1.90	14.59	18.42
Share of non-performing loans (%)	3.19	1.05	2.73	4.73
Realized default rate (% of performing loans)	1.89	0.90	1.36	3.43
Impairment allowances:				
Incurred losses	1.15	0.38	0.98	1.70
One-year expected losses	1.79	0.50	1.55	2.60
Lifetime expected losses	4.65	0.59	4.36	5.63
IFRS 9 allowances	2.67	0.62	2.38	3.66
Stage 1 allowances	0.24	0.05	0.22	0.33
Stage 2 allowances	1.28	0.21	1.18	1.63
Stage 3 allowances	1.15	0.38	0.98	1.70
IRB minimum capital requirement (CR)	8.15	0.07	8.14	8.19
IRB minimum capital requirement (CR) + CCB	10.69	0.09	10.68	10.74

The decomposition by stage shown for IFRS 9 reveals that allowances associated with NPLs followed by those associated with stage 2 loans are the ones that contribute the most to cross-state variation in impairment allowances. However, stage 3 loans are treated in the same way by all measures, so the differing volatilities of the various measures must come from the treatment of stage 1 loans (the same across EL^{1Y}, EL^{LT}, and EL^{IFRS9} but different in IL) and stage 2 loans (the same across EL^{LT} and EL^{IFRS9} but different in IL and EL^{1Y}) or from the cyclical shift of loans from stage 1 to stage 2 (under EL^{IFRS9}).

5.3 Impact on the cyclicality of P/L and CET1

Table 5 summarizes the impact of the various impairment measurement approaches on P/L and CET1 for an IRB bank. Confirming what one might expect after observing the volatility ranking of the impairment measures in Table 4, P/L is significantly more volatile under the more forward-looking EL^{LT} and EL^{IFRS9} than under EL^{1Y} or IL. EL^{IFRS9} (IL) is clearly the impairment measure producing a more (less) dissimilar P/L across aggregate states.

Table 5
P/L, CET1, dividends and recapitalizations under the aggregate-risk calibration
(IRB bank, % of mean exposures unless indicated)

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
P/L				
Unconditional mean	0.16	0.17	0.23	0.19
Conditional mean (s=1)	0.35	0.41	0.49	0.46
Conditional mean (s=2)	-0.46	-0.61	-0.66	-0.71
Standard deviation	0.34	0.43	0.51	0.50
CET1				
Unconditional mean	10.20	10.19	10.25	10.17
Conditional mean (s=1)	10.38	10.43	10.53	10.46
Conditional mean (s=2)	9.55	9.32	9.28	9.16
Standard deviation	0.76	0.76	0.71	0.77
Probability of paying dividends (%)				
Unconditional	49.53	51.79	56.38	53.93
Conditional (s=1)	64.20	67.11	73.07	69.89
Conditional (s=2)	0	0	0	0
Dividends, if positive				
Conditional mean (s=1)	0.35	0.36	0.42	0.38
Conditional mean (s=2)	–	–	–	–
Probability of having to recapitalize (%)				
Unconditional	2.34	2.86	2.34	3.41
Conditional (s=1)	0	0	0	0
Conditional (s=2)	10.26	12.50	10.22	14.94
Recapitalization, if positive				
Conditional mean (s=1)	–	–	–	–
Conditional mean (s=2)	0.42	0.40	0.34	0.38

The more forward-looking impairment measures are the ones that make the bank on average more CET1-rich in expansion states and less CET1-rich in contraction states, that is, the ones which make CET1 more procyclical in this sense. In any case, the reported quantitative differences are not huge in part because under our assumptions on the bank's management of its CET1, the range of variation of CET1 under any of the impairment measures is limited by the regulation-determined bands of the sS-rule described in equations (21) and (22). As explained above, the bank adjusts its CET1 to remain within those bands by paying dividends or raising new equity.

So a complementary way to assess the potential procyclicality associated with each impairment measure is to look at the frequency and size (conditional on them being strictly positive) of dividends and recapitalizations. Quite intuitively, under all measures we obtain that dividend distributions only occur (if at all) during expansion periods, while recapitalizations only occur (if at all) during contractions.

Relative to EL^{1Y}, the usage of EL^{IFRS9} implies an increase from 12% to 15% in the probability that the bank needs to be recapitalized during contractions (mirrored by a more modest increase from 67% to 70% in the probability of paying dividends during expansions).[24]

5.4 Effects of the arrival of a contraction

Using the same outlay as in Figure 2, Figure 4 shows the effects of the arrival of a contraction at t=0 (that is, the realization of s_0=2) after having spent a long enough period in the expansion state (that is, after having had s_t=1 for sufficiently many dates prior to t=0). From t=1 onwards the aggregate state follows the Markov chain calibrated in Table 3, thus making the trajectories followed by the variables depicted in Figure 4 stochastic. The figure depicts the average trajectories resulting from simulating 10000 paths.

The fact that the depicted trajectories are average trajectories is important for the right interpretation of Figure 4. For example, in panel D, the average trajectory of CET1 lies within the average bands of the sS-rule that determines its management, but this does not mean that the bank does not need to recapitalize (or does not pay dividends) after the initial shock. Actually, most of the actual trajectories either go up and touch the upper dividend-paying band (e.g. if the contraction ends and does not return) or go down and force the bank to recapitalize (e.g. if the contraction lasts long or another contraction follows soon after an initial recovery).

[24] These effects get, however, counterbalanced by the fact that, when strictly positive, the average size of the recapitalizations needed (and dividends paid) under EL^{IFRS9} is slightly lower than that under EL^{1Y}.

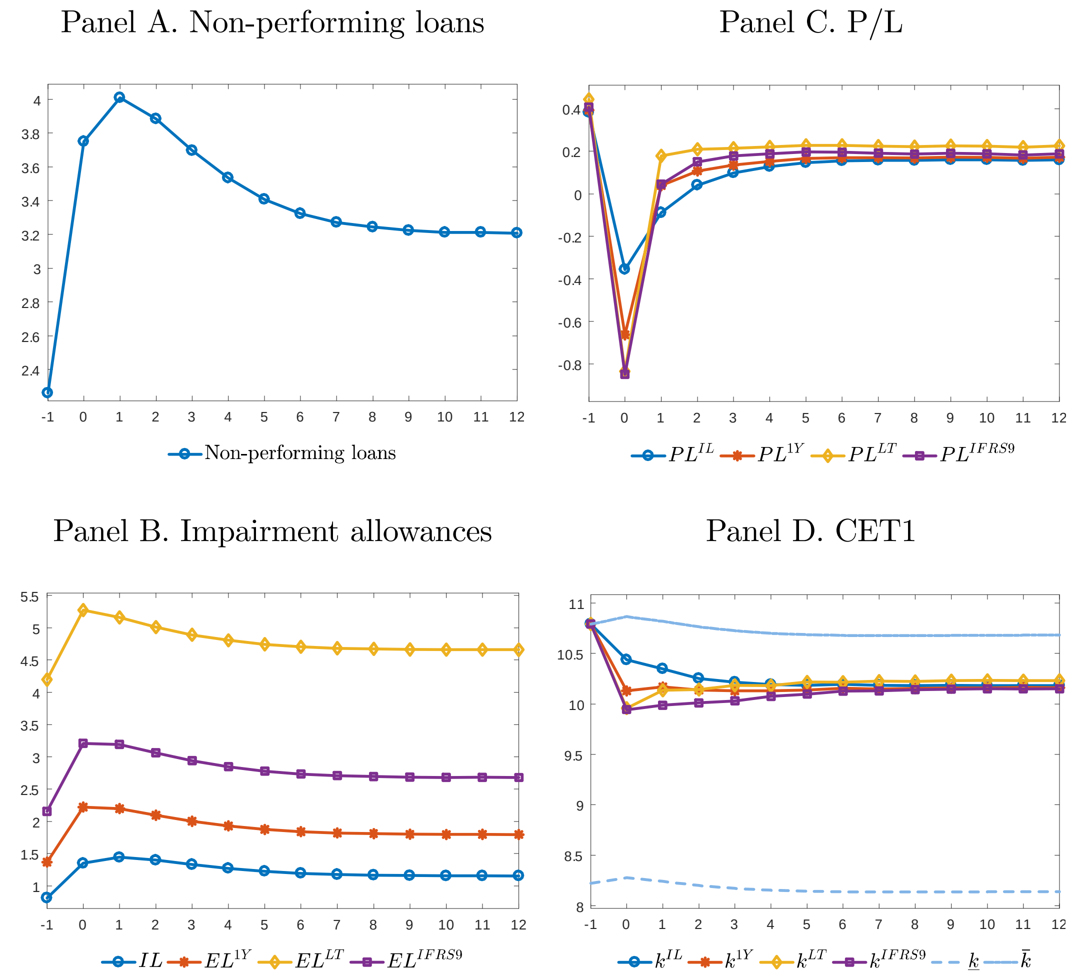

Figure 4. Effects of the arrival of a contraction
Average responses to the arrival of s=2 after long in s=1
(IRB bank, in % of average exposures).

To illustrate how average and actually realized trajectories differ, Figure 5 depicts 200 simulated trajectories for CET1 under EL^{1Y} and EL^{IFRS9}. Under the current calibration, it takes four consecutive years in the contraction state (s=2) for a bank under IFRS 9 to exhaust its CCB and require a recapitalization. In comparison, under the one-year expected loss approach, the CCB would only vanish after five years in contraction.

Intuitively, the closer the average trajectory for CET1 is to the lower band in Panel D of Figure 4, the more likely it is that the bank needs to raise new equity in the course of its recovery from the shock. Thus, as anticipated in Table 4, the probability that the bank needs to be recapitalized following the shock is higher under EL^{IFRS9} and EL^{LT} than under

EL^{1Y} or IL.

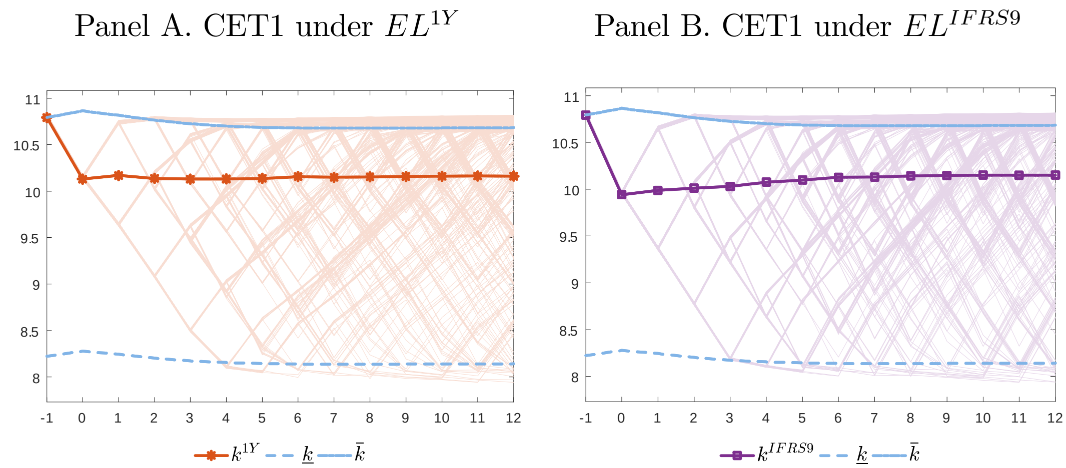

Figure 5. CET1 after the arrival of a contraction (IRB bank)
200 simulated trajectories of CET1 under EL^{1Y} and EL^{IFRS9}
in response to the arrival of s=2 after long in s=1
(IRB bank, in % of average exposures)

6 The case of SA banks

Capital requirements for banks following the standardized approach (SA banks) apply to exposures net of specific provisions and, hence, are sensitive to how those provisions get computed. So Table 6 below includes the same variables as Table 5 for an IRB banks together with details on the minimum capital requirement (CR) implied by each of the impairment measurement methods. Except CR and the implied size of a fully-loaded CCB, all the other variables in prior Table 4 are equally valid for IRB and SA banks.

The results in Table 6 are qualitatively very similar to those described for an IRB bank in Table 5, with some quantitative differences that are worth commenting. It turns out that for our calibration a SA bank holding exactly the same loan portfolio as an IRB bank would be able to support it with somewhat lower average levels of CET1 (between 48bps and 157 bps lower, depending on the impairment measurement method). Therefore, in a typical year, our SA bank features *de facto* slightly higher leverage levels and, hence, slightly higher interest expenses than its IRB counterpart. This explains why its P/L is slightly lower than that of an IRB bank. This difference explains most of the level differences observable in the remaining variables in Table 6.

Table 6
P/L, CET1, dividends and recapitalizations under SA capital requirements
(SA bank, % of mean exposures unless indicated)

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
P/L				
Unconditional mean	0.15	0.16	0.20	0.17
Conditional mean (s=1)	0.34	0.39	0.46	0.44
Conditional mean (s=2)	-0.46	-0.62	-0.69	-0.73
Standard deviation	0.34	0.43	0.51	0.50
Minimum CR				
Unconditional mean	7.72	7.57	6.88	7.36
Conditional mean (s=1)	7.72	7.56	6.88	7.35
Conditional mean (s=2)	7.74	7.58	6.89	7.37
Standard deviation	0.14	0.17	0.18	0.19
CET1				
Unconditional mean	9.70	9.50	8.68	9.23
Conditional mean (s=1)	9.88	9.76	8.97	9.54
Conditional mean (s=2)	9.04	8.61	7.67	8.19
Standard deviation	0.83	0.83	0.77	0.85
Probability of paying dividends (%)				
Unconditional	51.32	52.95	59.08	53.20
Conditional (s=1)	66.53	68.64	76.59	68.96
Conditional (s=2)	0	0	0	0
Dividends, if positive				
Conditional mean (s=1)	0.32	0.33	0.35	0.35
Conditional mean (s=2)	–	–	–	–
Probability of having to recapitalize (%)				
Unconditional	2.36	2.67	2.67	2.94
Conditional (s=1)	0	0	0	0
Conditional (s=2)	10.33	11.70	11.68	12.88
Recapitalization, if positive				
Conditional mean (s=1)	–	–	–	–
Conditional mean (s=2)	0.40	0.30	0.36	0.40

When comparing across impairment measurement methods within the SA bank case, the differences are very similar to those observed in Table 5 for IRB banks. The higher state-dependence of the more forward-looking measures explains the higher cross-state differences in CET1, dividends and probabilities of needing capital injections under such measures. As for IRB banks, the differences associated with IFRS 9 relative to either the incurred loss approach or the one-year expected loss approach are significant but not huge.

Table 7
SA banks vs IRB banks:
Highlighted differences
(% of mean exposures unless indicated)

	SA bank		IRB bank	
	IL	EL^{IFRS9}	EL^{1Y}	EL^{IFRS9}
P/L				
Unconditional mean	0.15	0.17	0.17	0.19
Standard deviation	0.34	0.50	0.43	0.50
Minimum CR				
Unconditional mean	7.72	7.36	8.15	8.15
Standard deviation	0.14	0.19	0.07	0.07
CET1				
Uncowith aggregate risk nditional mean	9.70	9.23	10.19	10.17
Standard deviation	0.83	0.85	0.76	0.77
Probability of paying dividends (%)				
Unconditional	51.32	53.20	51.79	53.93
Conditional on s=1	66.53	68.96	67.11	69.89
Dividends, if positive				
Mean conditional on s=1	0.32	0.35	0.36	0.38
Probability of having to recapitalize (%)				
Unconditional	2.36	2.94	2.86	3.41
Conditional on s=2	10.33	12.88	12.50	14.94
Recapitalization, if positive				
Mean conditional on s=2	0.40	0.40	0.40	0.38

To ease the comparison of the relevant differences across SA and IRB banks, Table 7 contains a selection of variables from previous Tables 4, 5 and 6. The selection is based on assuming that for a SA bank the relevant impairment allowances prior to the adoption of IFRS 9 are those of the incurred loss method, IL, while for a IRB bank are those of the one-year expected loss method, EL^{1Y}. The results point to IFRS 9 having an extremely similar quantitative impact across SA and IRB banks, both on the means and on the cyclical sensitivity of the relevant variables.

This is further confirmed by Figure 6, which shows the counterpart of Figure 5 for a bank operating under the SA approach. It depicts 200 simulated trajectories for CET1 under IL and EL^{IFRS9}. As in Figure 5, it takes four consecutive years in the contraction state (s=2) for a SA bank under IFRS 9 to exhaust its CCB and require a recapitalization, while under the incurred loss method, the CCB would only (roughly) vanish after five years in contraction.[25]

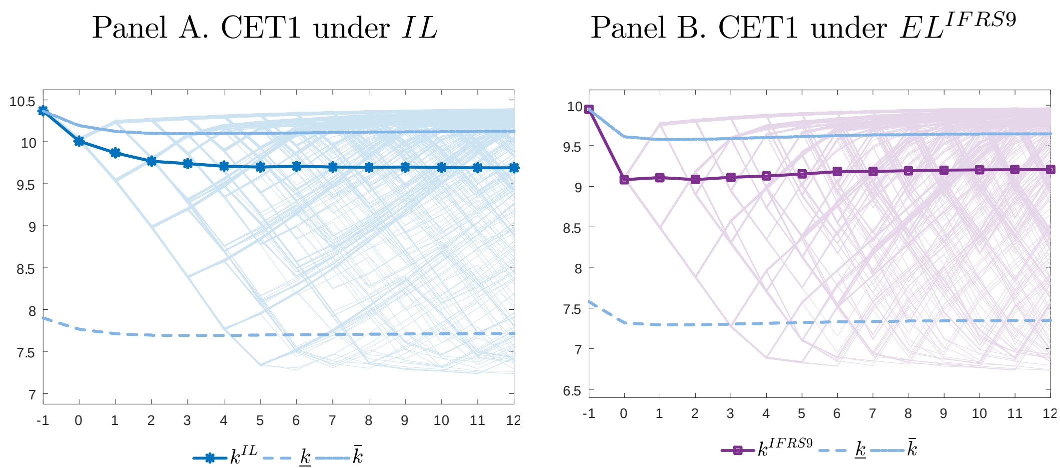

Figure 6. CET1 after the arrival of a contraction (SA bank)
200 simulated trajectories of CET1 under IL and EL^{IFRS9}
in response to the arrival of s=2 after long in s=1
(SA bank, in % of average exposures)

7 Extensions

7.1 Especially severe crises

In this section we explore whether the severity of crises and the potential anticipation of such a severity makes a difference with respect to the assessment of impairment measurement under IFRS 9 vis-a-vis less forward-looking measures. To keep our graphs readable, we focus the attention on IRB banks and compare the IFRS 9 approach with just one of the

[25]In this case, the depicted dashed lines that delimit the band within which CET1 evolves are averages across simulated trajectories, since the relevant sizes of CR and CR plus a fully-loaded CCB depend on the size of the corresponding allowances.

alternatives, the one-year expected loss approach, which under our formulation is similar to the current regulatory approach to loan loss provisioning for IRB banks.

7.1.1 Unanticipatedly long crises

We first explore what happens with the dynamic responses analyzed in the benchmark calibration with aggregate risk when we condition them on the realization of the contraction state s=2 for four consecutive periods starting from t=0. So, as in the analysis in prior Figure 4, we assume that the bank starts at t=–1 with the portfolio and impairment allowances resulting from having been long enough in the expansion state (s=1) and that at t=0 the aggregate state switches to contraction (s=2).

In Figure 7 we compare the average response trajectories already shown in Figure 4 (where from t=1 onwards the aggregate state evolves stochastically according to the Markov chain calibrated in Table 3) with trajectories conditional on remaining in state s=2 for at least up to date t=3 (four years).[26]

When a crisis turns unexpectedly long, the largest differential impact of EL^{IFRS9} relative to EL^{1Y} still happens in the first year of the crisis (t=0), as EL^{IFRS9} front-loads the expected beyond-one-year losses of the stage 2 loans. In years two to four of the crisis (t=1,2,3) the differential impact of IFRS 9 (vs. one-year) expected losses on P/L falls before it switches sign (after t=5). In the first years of the crisis, EL^{IFRS9} leaves CET1 closer to the recapitalization band and in the fourth year (t=3), the duration of the crisis forces the bank to recapitalize only under EL^{IFRS9}. On the other hand, EL^{IFRS9} supports a quicker recovery of profitability and, hence, CET1 after t=5.

[26] In the conditional trajectories, the aggregate state is again assumed to evolve according to the calibrated Markov chain from t=4 onwards.

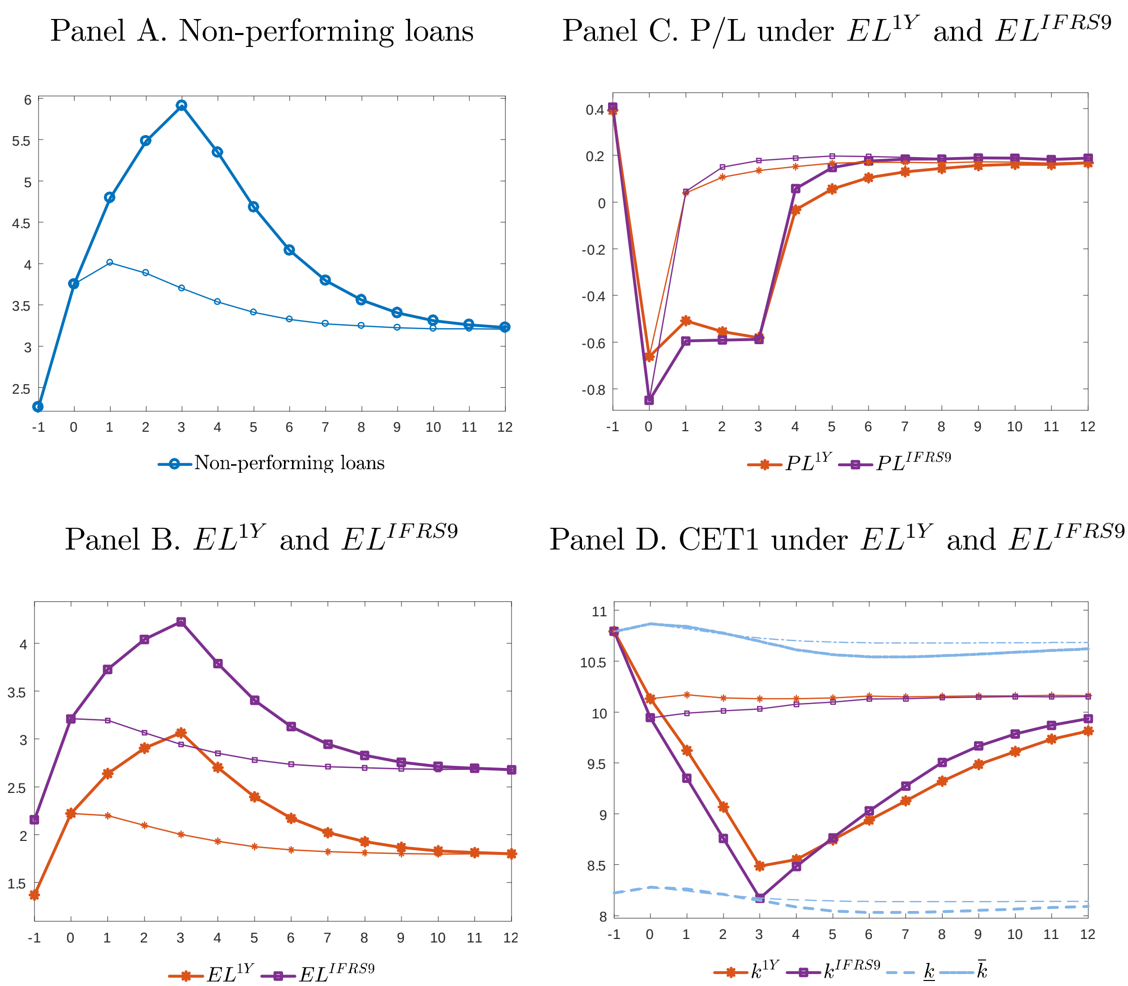

Figure 7. Unanticipatedly long crises
Average responses to the arrival of s=2 when when the contraction is unanticipatedly "long" (thick lines) rather than "average" (thin lines) (IRB bank, in % of average exposures)

7.1.2 Anticipatedly long crises

We now turn to the case in which long crises can be detected to be such from the beginning. To study this case, we extend the model to add a third aggregate state that describes "long crises" (s=3) as opposed to "short crises" (s=2) or "expansions" (s=1). To streamline the analysis, we make s=2 and s=3 to have exactly the same impact on credit risk parameters as prior s=2 in Table 3, and keep the impact of s=1 on credit risk parameters also exactly the same as in Table 3. The only difference between states s=2 and s=3 is their persistence, which determines the average time it takes for a crisis period to end. Specifically, we consider the following transition probability matrix for the aggregate state:

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.8520 & 0.6348 & 0.250 \\ 0.1221 & 0.3652 & 0 \\ 0.0259 & 0 & 0.750 \end{pmatrix}, \tag{26}$$

which implies an average duration of 4 years for long crises (s=3), 1.6 years for short crises (s=2), and the same duration as in our benchmark calibration for expansion periods (s=1). The parameters in (26) are calibrated to make s=3 to occur with an unconditional frequency of 8% (equivalent to suffering an average of two long crises per century) and to keep the unconditional frequency of s=1 at the same 77% as in our benchmark calibration.

In Figure 8 we compare the average response trajectories that follow the entry in state s=2 (thin lines) or state s=3 (thick lines) after having spent a sufficiently long period in state s=1. So the figure illustrates the average differences between entering a "normal" short crisis or a "less frequent" long crisis at t=0. Notice that both EL^{1Y} and EL^{IFRS9} behave differently across short and long crisis from the very first period, since even the one-year ahead loss projections behind EL^{1Y} factor in the lower probability of a recovery at t=1 under s=3 than under s=2. But EL^{IFRS9} additionally takes into account the losses further into the future associated with the stage 2 loans. Hence the differential rise on impact experienced by EL^{IFRS9} is higher than that experienced by EL^{1Y}. This also explains a larger differential initial impact on P/L and CET1. As a result, when entering in an anticipatedly long crisis, EL^{IFRS9} pushes CET1 closer to the recapitalization band and the difference with respect to EL^{1Y} increases. Quantitatively, however, the effect on CET1 is still moderate, consuming on impact less than half of the fully loaded CCB. On the other hand, later in the long crisis, EL^{IFRS9} leads, on average, to a quicker recovery of profitability and CET1 than EL^{1Y}.

As a quantitative summary of the implications of having anticipatedly long crisis, the following table reports the unconditional yearly probabilities of the bank needing equity injections, under each of the compared impairment measures, in the baseline model with aggregate risk and in the current extension:

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
Baseline model	2.34%	2.86%	2.34%	3.41%
Model with anticipatedly long crises	3.28%	3.78%	4.23%	4.52%

7.2 Better foreseeable crises

We now consider the case in which some crises can be foreseen one year in advance. Akin to the treatment of long crises in the previous subsection, we formalize this by introducing a third aggregate state s=3 which describes normal or expansion states in which a crisis (transition to state s=2) is expected in the next year with a larger than usual probability. So we make s=3 identical to s=1 in all respects (that is, the way it affects the PDs, rating migration probabilities, and LGDs of the loans, et cetera) except in the probability of switching to aggregate state $s' = 2$ in the next year.

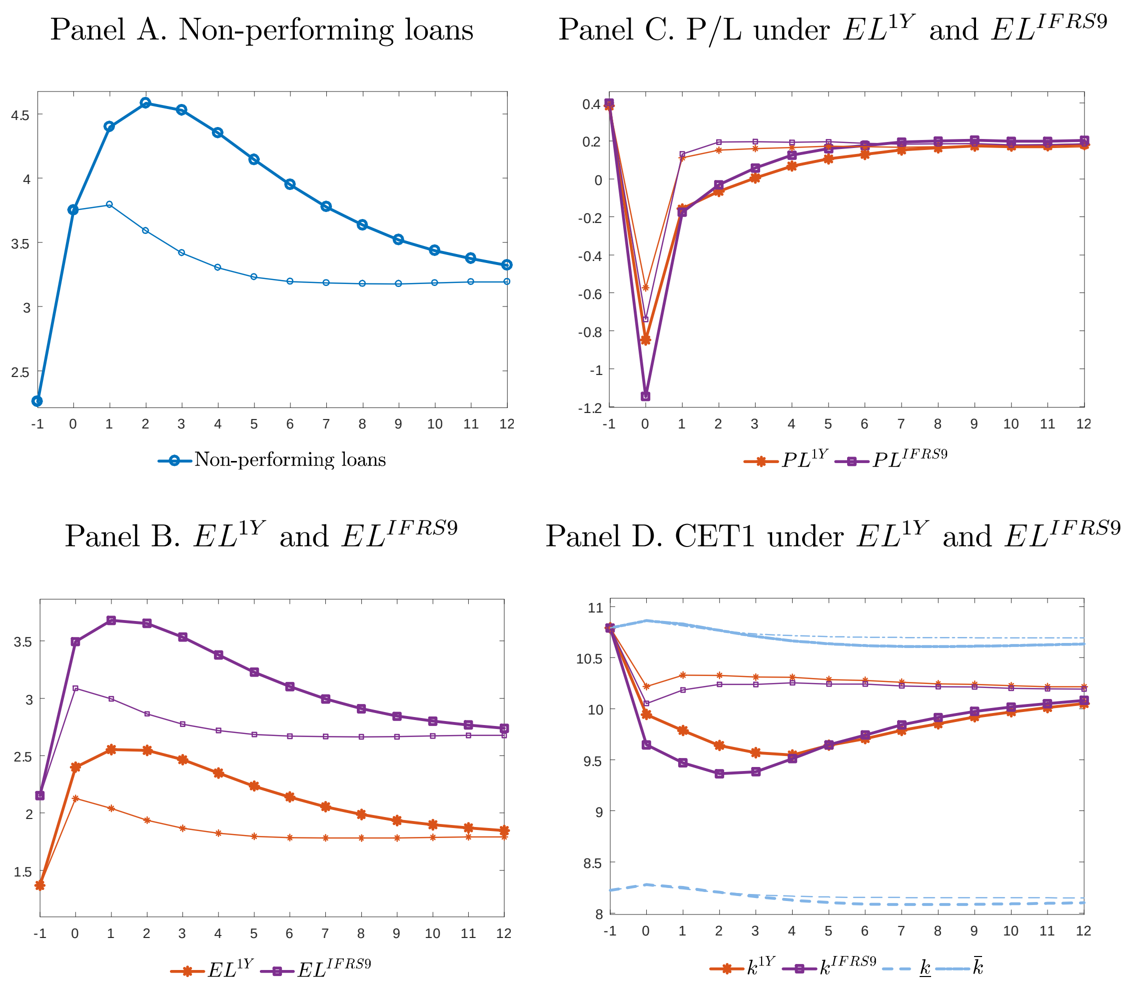

Figure 8. Anticipatedly long crises
Average responses to the arrival of a contraction at t=0 when it is anticipated to be "long" (s'=3, thick lines) rather than "normal" (s'=2, thin lines) (IRB bank, in % of average exposures)

To streamline the analysis we look at the case in which s=3 is followed by s'=1 with probability one and assume that half of the crisis are preceded by $s = 3$ (while the other half are preceded, as before, by $s = 1$, which means that they are not "seen coming"). Adjusting the transition probabilities to imply the same relative frequencies and expected durations of non-crisis versus crisis periods as the baseline calibration in Table 3, the matrix of state transition probabilities used for this exercise is

$$\begin{pmatrix} p_{11} & p_{12} & p_{13} \\ p_{21} & p_{22} & p_{23} \\ p_{31} & p_{32} & p_{33} \end{pmatrix} = \begin{pmatrix} 0.8391 & 0.5 & 0 \\ 0.0740 & 0.5 & 1 \\ 0.0869 & 0 & 0 \end{pmatrix}.$$

The thick lines in Figure 9 show the average response paths to the arrival of the pre-crisis state s'=3 at t=–1 after having spent a long time in the normal state s=1. We compare EL^{1Y} and EL^{IFRS9} and include, using thin lines, the results of the baseline model (regarding the arrival of s'=2 at t=0 after having been for long in s=1). The results confirm the intuition that better anticipating the arrival of a crisis helps to considerably smooth away its impact on impairment allowances, P/L, and CET1.

Finally, as in the previous extension, the following table reports the unconditional yearly probabilities of the bank needing equity injections, under each of the compared impairment measures, in the baseline model with aggregate risk and in the current extension. Indeed, better anticipated crises imply a lower yearly probability that the bank needs an equity injection:

	IL	EL^{1Y}	EL^{LT}	EL^{IFRS9}
Baseline model	2.34%	2.86%	1.62%	3.34%
Model with better foreseeable crises	1.84%	1.99%	1.54%	2.66%

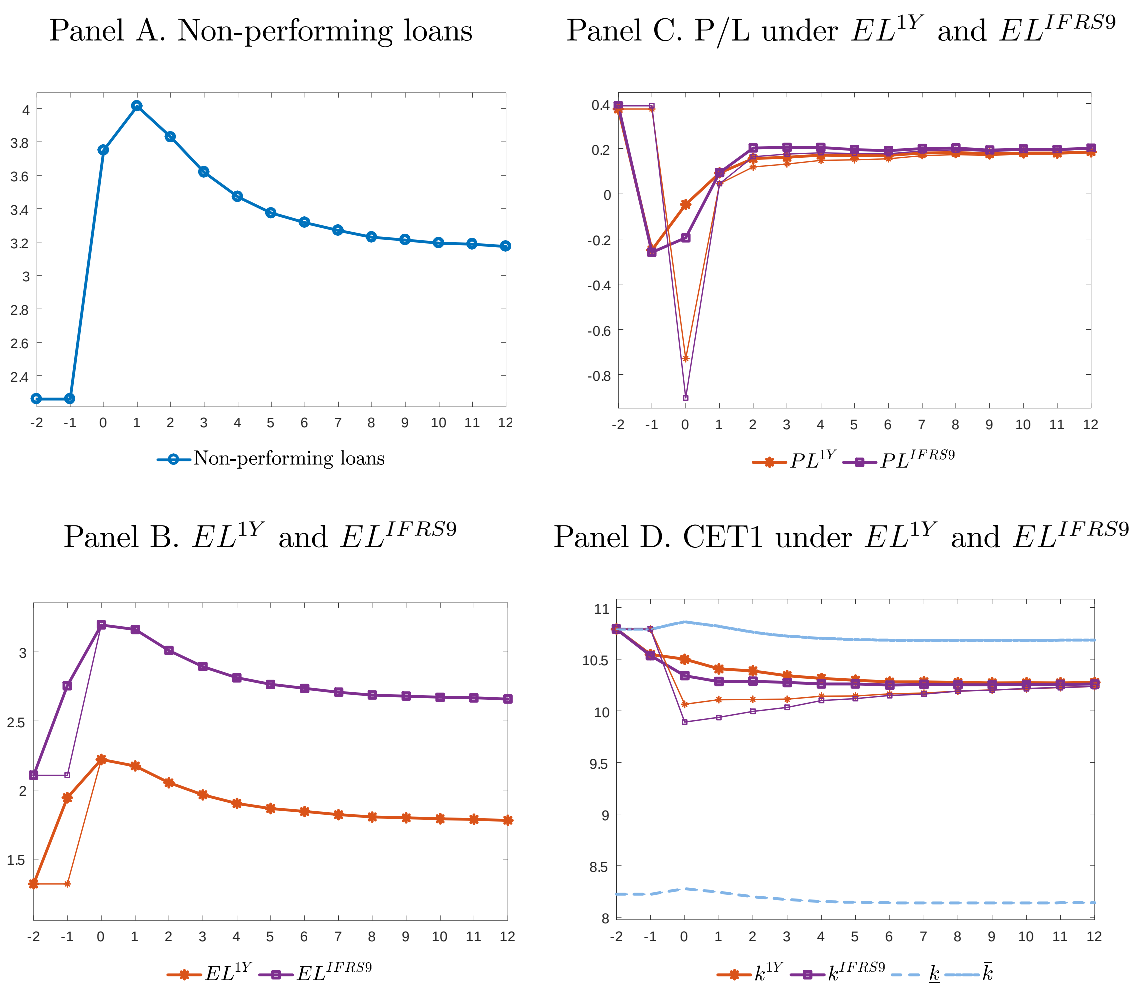

Figure 9. Better foreseeable crises
Average responses to the arrival of pre-crisis state at t=–1 after long in s=1 (thick lines). Thin lines describe the arrival of s=2 at t=0 in the baseline model (IRB bank, in % of average exposures)

7.3 Other possible extensions

In this section we briefly sketch additional extensions that the model might accommodate at some cost in terms of notational, computational, and calibration complexity.

Multiple standard and substandard ratings Adding more rating categories within the broader standard and substandard categories would essentially imply expanding the dimensionality of the vectors and matrices described in the baseline model and its aggregate-risk extension. If loans are assumed to be originated in more than just one category, the need to keep track of the (various) contractual interest rates for discounting purposes would oblige to expand the dimensionality of the model further. Instead of doing this, one equivalent and possibly less notationally cumbersome possibility would be to consider as many portfolios as different-at-origination loans, and to aggregate impairment allowances and the implications for P/L and CET1 across them.

Relative criterion for credit quality deterioration This extension would be a natural further development of the previous one. Under IFRS 9, the shift to the life-time approach for a given loan is supposed to be applied not when an "absolute" substandard rating is attained but when the deterioration with respect to the rating at origination is significant in "relative" terms, e.g. because the rating has fallen by more than two or three notches. This distinction is relevant if operating under a ratings' scale finer than the one we have used in our analysis. As in the case with multiple standard and substandard ratings just described, keeping the analysis recursive under the relative criterion for treating loans as "stage 1" or "stage 2" loans in IFRS 9 would require considering as many portfolios as different-at-origination loan ratings and to write expressions for impairment allowances that impute lifetime expected losses to the components of each portfolio whose current rating is lower enough than the initial one.

8 Macroprudential implications

What are the implications of these results regarding the potential procyclical effects linked to the various impairment measures? Is the measure associated with IFRS9 more procyclical than its predecessors? Answering these questions is difficult. Even in the absence of offsetting regulatory filters or sufficient excess capital buffers, a fall in CET1 that reduces the bank's CCB (and hence forces it to cancel its dividends) or even leads to the need for equity issuance in order to keep complying with the minimum capital requirements does not necessarily imply that credit supply will contract. It will depend on the bank's dislike for cancelling dividends and, if the CCB is lost, on how quickly or cheaply the bank can raise new capital.

Our simulations are produced as-if there were no concerns or imperfections on these fronts. Otherwise, the bank might be induced to reduce its lending. If this process occurs at an economy wide level (e.g. in response to an aggregate shock), the contractive effects on aggregate credit supply might be significant, potentially causing negative second round effects on the system (e.g. by weakening aggregate demand or damaging interfirm credit chains), eventually producing larger default rates on surviving loans.

These feedback effects, although theoretically and empirically difficult to assess, are at the heart of the motivation for the macroprudential approach to financial regulation.[27] As in discussions around the potential procyclical effects of Basel capital requirements (Kashyap and Stein, 2004, and Repullo and Suarez, 2013), whether IFRS 9 ends up adding procyclicality to the system or not will depend on multiple factors. For example, even if it causes a contraction in credit supply when a negative shock hits the economy, such contraction might be lower than the contraction in credit demand, which can also be negatively affected by the shock. Moreover, banks may react to IFRS 9 by having larger voluntary capital buffers in the first place. Besides, the negative effects of an additional contraction in loan supply might be counterbalanced by the advantages of an earlier recognition of loan losses (e.g. by precluding forbearance or the continuation of dividend payments during the initial stages of a crisis), including the possibility that they allow for a quicker recovery of banks' health.

Despite all these caveats, recent evidence (including Mésonnier and Monks, 2015, Gropp et al., 2016, and Jiménez et al., 2017) suggests that sudden increases in capital requirements or other regulatory buffers (or, similarly, falls in available regulatory capital) tend to be accommodated by banks with reductions in risk-weighted assets, most typically bank lending, causing significant effects on the real economy. While the size of the additional procyclical losses of regulatory capital implied by our results is not alarming, it is significant enough to warrant further macroprudential attention.

Fortunately there is a broad range of policies that might help address the procyclical effects of IFRS 9 if deemed necessary. One possibility is to rely on the existing regulatory buffers and, specifically, on the countercyclical capital buffer (CCyB), possibly after a suitable

[27] As put by Hanson, Kashyap and Stein (2011, p. 5), "in the simplest terms, one can characterize the macroprudential approach to financial regulation as an effort to control the social costs associated with excessive balance sheet shrinkage on the part of multiple financial institutions hit with a common shock."

revision of its guidance. The national macroprudential authorities might proactively use the CCyB with the purpose of off-setting undesirable credit supply effects. This would involve setting the CCyB at a level above zero in expansionary or normal times so as to have the capacity to partly or fully release it if and when the change in aggregate conditions leads to a sudden increase in impairment allowances. This macroprudential tool might be combined with the use of internal and external stress tests as a means to gauge the importance of the variation in impairment allowances associated with adverse scenarios, guarantee the sufficiency of the micro and macroprudential buffers, and allow for remedial policy action if required.

9 Concluding remarks

We have described a simple recursive model for the assessment of the level and cyclical implications of credit impairment loss measurement under IFRS 9. We have calibrated the model to represent a portfolio of corporate exposures of a EU bank. We have compared the level and dynamic responses to negative shocks of alternative impairment measurement approaches: the current incurred loss approach, the one-year expected loss approach (used to establish the regulatory provisions of IRB banks), the lifetime expected loss approach (which is the one planned by FASB for the US), and the mixed-horizon expected loss approach of IFRS 9.

Our results suggest that IFRS 9 (and, similarly, the lifetime expected loss approach) will imply more sudden rises in impairment allowances when the cyclical position of the economy switches from expansion to contraction (or if banks experience a shock that sizably damages the credit quality of their loan portfolios). This implies that P/L and, absent regulatory filtering, CET1 will decline more severely at the start of those episodes.

While an early and decisive recognition of forthcoming losses may have significant advantages (e.g. in terms of transparency, market discipline, inducing prompt supervisory intervention, et cetera), it may also imply, via its effects on regulatory capital, a loss of lending capacity for banks right at the beginning of a contraction (or right in the aftermath of a negative credit-quality shock), potentially contributing, through feedback effects, to its severity. With this concern in mind, the quantitative results of the paper suggest that the arrival of an average recession might imply an on-impact loss of CET1 equivalent to one third of the fully-loaded capital conservation buffer of the analyzed bank. While this loss is larger than under the one-year expected loss approach of current regulatory provisions, its distance from the amount that would deplete the fully-loaded CCB is tranquilizing. Notwithstanding, it would be adequate for macroprudential authorities to keep an eye on developments in this front (e.g. by relying on stress testing) and to remain ready to take compensatory measures (e.g. the release of the CCyB), in case of need.

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Appendices

A Calibration details

A.1 Migration and default rates for our two non-default states

We calibrate the migration and default probabilities of our two non-default loan categories using S&P rating migration data referred to a finer rating partition. Specifically, let the 7×7 matrix $\widetilde{A}$ describe yearly migrations across the seven non-default ratings in the main S&P classification: AAA, AA, A, BBB, BB, B and CCC/C, respectively. Under our convention, each element $\tilde{a}_{ij}$ of such matrix denotes a loan's probability of migrating to S&P rating i from S&P rating j, and the yearly probability of default corresponding to S&P rating j can be found as $\widetilde{PD}_j = 1 - \sum_{i=1}^{7} \tilde{a}_{ij}$.[28] We obtain $\widetilde{A}$ by averaging the yearly matrices provided by S&P global corporate default studies covering the period 1981-2015:

$$\widetilde{A} = \begin{pmatrix} 0.8960 & 0.0054 & 0.0005 & 0.0002 & 0.0002 & 0.0000 & 0.0007 \\ 0.0967 & 0.9073 & 0.0209 & 0.0022 & 0.0008 & 0.0006 & 0.0000 \\ 0.0048 & 0.0798 & 0.9161 & 0.0463 & 0.0034 & 0.0026 & 0.0022 \\ 0.0010 & 0.0056 & 0.0557 & 0.8930 & 0.0626 & 0.0034 & 0.0039 \\ 0.0005 & 0.0007 & 0.0044 & 0.0465 & 0.8343 & 0.0618 & 0.0112 \\ 0.0003 & 0.0009 & 0.0017 & 0.0082 & 0.0809 & 0.8392 & 0.1390 \\ 0.0006 & 0.0002 & 0.0002 & 0.0013 & 0.0079 & 0.0432 & 0.5752 \end{pmatrix}, \tag{27}$$

which implies

$$\widetilde{PD}^T = (0.0000, 0.0002, 0.0005, 0.0023, 0.0100, 0.0493, 0.2678).$$

In order to calibrate our model, we want to collapse the above seven-state Markov process into the two-state one specified in our model. We want to obtain its 2×2 transition probability matrix, which we denote A, and the implied probabilities of default in each state, $PD_j = 1 - \sum_{i=1}^{2} a_{ij}$ for j=1,2. To map the seven-state process into the two-state one, we assume that the S&P states 1 to 5 (AAA, AA, A, BBB, BB) correspond to our state 1 and S&P states 6 to 7 (B, CCC/C) to our state 2. We additionally assume that all the loans originated by the bank belong to the BB category, so that the vector representing the entry of new loans in steady state under the S&P classification is $\tilde{e}^T = (0,0,0,0,1,0,0)$. Under these assumptions, we produce an average PD for the steady state portfolio of 1.88%, slightly below the 2.5% average PD on non-defaulted exposures of reported by EBA (2013, Figure 12) for the period 2009h1-2012h2 for a sample of EU banks using the IRB approach.

[28] We have reweighted the original migration rates in S&P matrices to avoid having "non-rated" as a possible migration.

The steady state portfolio under the S&P classification can be found as $z^* = [I_{7\times 7} - \widetilde{M}]^{-1}\widetilde{e}$, where the matrix $\widetilde{M}$ has elements $\widetilde{m}_{ij} = (1-\delta_j)\widetilde{a}_{ij}$ and δ_j is the independent probability of a loan rated j maturing at the end of period t. For the calibration we set δ_j=0.20 across all categories, so that loans have an average maturity of 5 years. The "collapsed" steady state portfolio x^* associated with z^* has $x_1^* = \sum_{j=1}^{5} z_j^*$ and $x_2^* = \sum_{j=6}^{7} z_j^*$.

For the collapsed portfolio, we construct the 2×2 transition matrix M (that accounts for loan maturity) as

$$M = \begin{pmatrix} \frac{\sum_{j=1}^{5}\sum_{i=1}^{5}\widetilde{m}_{ij}z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=1}^{5}\widetilde{m}_{ij}z_j^*}{x_2^*} & 0 \\ \frac{\sum_{j=1}^{5}\sum_{i=6}^{7}\widetilde{m}_{ij}z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=6}^{7}\widetilde{m}_{ij}z_j^*}{x_2^*} & 0 \\ (1-\delta_3/2)PD_1 & (1-\delta_3/2)PD_2 & (1-\delta_3) \end{pmatrix}, \tag{28}$$

where the probabilities of default for the collapsed categories are found as

$$PD_1 = \frac{\sum_{j=1}^{5}\widetilde{PD}_j z_j^*}{x_1^*},$$

and

$$PD_2 = \frac{\sum_{j=6}^{7}\widetilde{PD}_j z_j^*}{x_2^*}.$$

Putting it in words, we find the moments describing the dynamics of the collapsed portfolio as weighted averages of those of the original distribution, with the weights determined by the steady state composition of the collapsed categories in terms of the initial categories.

A.2 Calibrating defaulted loans' resolution rate

The yearly probability of resolution of non-performing loans δ_3 is calibrated to match the 5% average probability of default including defaulted exposures ($PDID$) that EBA (2013, Figure 10) reports for 2008h2. In the model the value of such probability in steady state can be computed as

$$PDID = \frac{PD_1 x_1^* + PD_2 x_2^* + x_3^*}{\sum_{j=1}^{3} x_j^*}.$$

Solving for x_3^* we find

$$x_3^* = \frac{PD_1 x_1^* + PD_2 x_2^* - (x_1^* + x_2^*)PDID}{PDID - 1}. \tag{29}$$

Importantly, the dynamic system in (1) allows us to compute x_1^* and x_2^* independently from δ_3, so the law of motion of NPLs evaluated at the steady state implies

$$x_3^* = (1-\delta_3/2)PD_1 x_1^* + (1-\delta_3/2)PD_2 x_2^* + (1-\delta_3)x_3^*$$

or

$$\delta_3 = \frac{2(PD_1 x_1^* + PD_2 x_2^*)}{PD_1 x_1^* + PD_2 x_2^* + 2x_3^*}. \tag{30}$$

Finally, we can evaluate (30) using x_1^*, x_2^* and the value of x_3^* found in (29).

A.3 State contingent migration matrices

In the model described in Appendix B, we capture aggregate risk through an aggregate state variable $s_t \in \{1, 2\}$ that follows a Markov chain with a time-invariant transition matrix. We calibrate the state contingent migration matrices $M(1)$ and $M(2)$ of such a version of the model following a procedure analogous to that leading to obtain M in (28) but starting from state-contingent versions, $\widetilde{A}(1)$ and $\widetilde{A}(2)$, of the 7×7 migration matrix $\widetilde{A}$ in (27). As described in A.1, we can go from each $\widetilde{A}(s)$ to the maturity adjusted matrix $\widetilde{M}(s)$ with elements $\widetilde{m}_{ij}(s) = (1 - \delta_j)\widetilde{a}_{ij}$ and then find the elements of $M(s)$ as weighted averages of the elements of $\widetilde{M}(s)$. To keep things simple, we use the same unconditional weights as in (28) implying

$$M(s) = \begin{pmatrix} \frac{\sum_{j=1}^{5}\sum_{i=1}^{5}\widetilde{m}_{ij}(s)z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=1}^{5}\widetilde{m}_{ij}(s)z_j^*}{x_2^*} & 0 \\ \frac{\sum_{j=1}^{5}\sum_{i=6}^{7}\widetilde{m}_{ij}(s)z_j^*}{x_1^*} & \frac{\sum_{j=6}^{7}\sum_{i=6}^{7}\widetilde{m}_{ij}(s)z_j^*}{x_2^*} & 0 \\ (1-\delta_3(s)/2)PD_1(s) & (1-\delta_3(s)/2)PD_2(s) & (1-\delta_3(s)) \end{pmatrix}$$

where

$$PD_1(s) = \frac{\sum_{j=1}^{5}\widetilde{PD}_j(s)z_j^*}{x_1^*},$$

$$PD_2(s) = \frac{\sum_{j=6}^{7}\widetilde{PD}_j(s)z_j^*}{x_2^*},$$

with $\widetilde{PD}_j(s) = 1 - \sum_{i=1}^{7}\tilde{a}_{ij}(s)$.

We calibrate $\widetilde{A}(1)$ and $\widetilde{A}(2)$ exploring the business cycle sensitivity of S&P yearly migration matrices previously averaged to find $\widetilde{A}$. We identify state s=1 with normal or expansion years and s=2 with crisis or contraction years. We use NBER start recession years to identify the entry in state s=2 and assume that each of the contractions observed in the period 1981-2015 lasted exactly two years. This is consistent with the NBER dating of US recessions except for the recession started in 2001, to which the NBER attributes a duration of less than one year. However, the behavior of corporate ratings migrations and defaults around such recession does not suggest it was shorter for our purposes than the other three. To illustrate this, Figure A1 depicts the time series of two of the elements of the yearly default rates $\widetilde{PD}_j$ and migration matrices $\widetilde{A}$ whose cyclical behavior is more evident: (i) the default rate among BB exposures ($\widetilde{PD}_5$) and (ii) the migration rate from a B rating to a CCC/C rating ($\tilde{a}_{7,6}$). Year 2002 emerges clearly as a year of marked deterioration in credit quality among exposures rated BB and B.

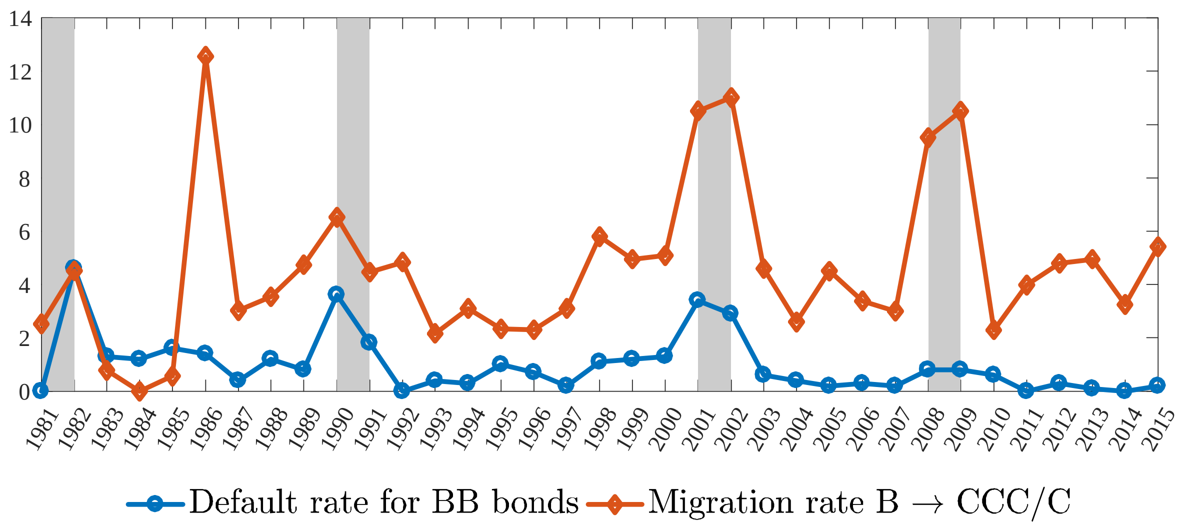

Figure A1. Sensitivity of default and migrations rates to aggregate states
Selected yearly S&P default and downgrading rates. Grey bars identify 2-year periods following the start of NBER recessions

In light of this, we estimate $\widetilde{A}(2)$ by averaging the yearly migration matrices of years 1981, 1982, 1990, 1991, 2001, 2002, 2008 and 2009, and $\widetilde{A}(1)$ by averaging all the remaining ones. This leads to

$$\widetilde{A}(1) = \begin{pmatrix} 0.8923 & 0.0057 & 0.0005 & 0.0002 & 0.0002 & 0.0000 & 0.0000 \\ 0.1012 & 0.9203 & 0.0209 & 0.0023 & 0.0007 & 0.0003 & 0.0000 \\ 0.0039 & 0.0668 & 0.9228 & 0.0500 & 0.0036 & 0.0025 & 0.0027 \\ 0.0010 & 0.0058 & 0.0495 & 0.8939 & 0.0668 & 0.0036 & 0.0043 \\ 0.0007 & 0.0002 & 0.0040 & 0.0429 & 0.8484 & 0.0679 & 0.0117 \\ 0.0000 & 0.0009 & 0.0020 & 0.0084 & 0.0680 & 0.8511 & 0.1548 \\ 0.0000 & 0.0002 & 0.0001 & 0.0009 & 0.0059 & 0.0360 & 0.5860 \end{pmatrix},$$

implying

$$\widetilde{PD}(1)^T = (0.0000, 0.0001, 0.0002, 0.0014, 0.0063, 0.0386, 0.2405),$$

and

$$\widetilde{A}(2) = \begin{pmatrix} 0.9087 & 0.0044 & 0.0003 & 0.0005 & 0.0002 & 0.0000 & 0.0030 \\ 0.0786 & 0.8632 & 0.0209 & 0.0014 & 0.0013 & 0.0017 & 0.0000 \\ 0.0077 & 0.1237 & 0.8936 & 0.0340 & 0.0026 & 0.0027 & 0.0009 \\ 0.0010 & 0.0050 & 0.0767 & 0.8899 & 0.0482 & 0.0028 & 0.0024 \\ 0.0000 & 0.0022 & 0.0057 & 0.0587 & 0.7865 & 0.0411 & 0.0095 \\ 0.0013 & 0.0007 & 0.0008 & 0.0076 & 0.1245 & 0.7988 & 0.0858 \\ 0.0027 & 0.0002 & 0.0006 & 0.0025 & 0.0143 & 0.0676 & 0.5389 \end{pmatrix},$$

implying

$$\widetilde{PD}(2)^T = (0.0000, 0.0005, 0.0014, 0.0054, 0.0224, 0.0853, 0.3596).$$

Finally, we set p_{12} =Prob$(s_{t+1} = 1|s_t = 2)$ equal to 0.5 so that contractions have an expected duration of two years, and p_{21} =Prob$(s_{t+1} = 2|s_t = 1)$ equal to 0.148 so that expansion periods have the same average duration as the ones observed in our sample period, (35-8)/4=6.75 years.

B The model with aggregate risk

In this appendix we present the equations of the benchmark model with aggregate risk. We capture the latter by introducing an aggregate state variable that can take two values $s_t \in \{1, 2\}$ at each date t and follows a Markov chain with time-invariant transition probabilities $p_{s's}$ =Prob$(s_{t+1} = s'|s_t = s)$. The approach can be trivially generalized to deal with a larger number of aggregate states.

In order to measure expected losses corresponding to default events in any future date t, we have to keep track of the aggregate state in which the loans existing at t were originated, z=1,2, the aggregate state at time t, s=1, 2, and the credit quality or rating of the loan at t, j=1, 2, 3. Thus it is convenient to describe (stochastic) loan portfolios held at any date t as vectors of the form

$$y_t = \begin{pmatrix} x_t(1,1,1) \\ x_t(1,1,2) \\ x_t(1,1,3) \\ x_t(1,2,1) \\ x_t(1,2,2) \\ x_t(1,2,3) \\ x_t(2,1,1) \\ x_t(2,1,2) \\ x_t(2,1,3) \\ x_t(2,2,1) \\ x_t(2,2,2) \\ x_t(2,2,3) \end{pmatrix}, \qquad (31)$$

where component $x_t(z, s, j)$ denotes the measure of loans at t that were originated in aggregate state z, are in aggregate state s and have rating j.[29]

Our assumptions about the evolution and payoffs of the loans between any date t and $t+1$ are the following. Loans rated j=1, 2 at t mature at $t + 1$ with probability $\delta_j(s')$, where s' denotes the aggregate state at $t+1$ (unknown at date t). In the case of non-performing loans

[29] Along a specific history (or sequence of aggregate states), for any z and j, the value of $x_t(z, s, j)$ will equal 0 whenever $s_t \neq s$.

(j=3), $\delta_3(s')$ represents the independent probability of a loan being resolved, in which case it pays back a fraction $1-\widetilde{\lambda}(s')$ of its unit principal and gets extinguished. Conditional on s', each loan rated j=1, 2 at t that matures at $t+1$ defaults independently with probability $PD_j(s')$, being resolved within the period with probability $\delta_3(s')/2$ or entering the stock of non-performing loans (j=3) with probability $1-\delta_3(s')/2$. Maturing loans that do not default pay back their principal of one plus the contractual interest c_z, established at origination.

Conditional on s', each loan rated j=1, 2 at t that does not mature at $t+1$ goes through one of the following exhaustive possibilities:

1. Default, which occurs independently with probability $PD_j(s')$, and in which case one of two things can happen: (i) it is resolved within the period with probability $\delta_3(s')/2$; or (ii) it enters the stock of non-performing loans (j=3) with probability $1-\delta_3(s')/2$.

2. Migration to rating $i \neq j$ (i=1,2), in which case it pays interest c_z and continues for one more period; this occurs independently with probability $a_{ij}(s')$.

3. Staying in rating j, in which case it pays interest c_z and continues for one more period; this occurs independently with probability

$$a_{jj}(s') = 1 - a_{ij}(s') - PD_j(s').$$

B.1 Portfolio dynamics under aggregate risk

Under aggregate risk, the dynamics of the loan portfolio between any dates t and $t+1$ is no longer deterministic but driven by the realization of the aggregate state variable at $t+1$, s_{t+1}. To describe the dynamics of the system compactly, let the binary variable $\xi_{t+1}=1$ if $s_{t+1}=1$ and $\xi_{t+1}=0$ if $s_{t+1}=2$. The dynamics of the system can be described as:

$$y_{t+1} = G(\xi_{t+1})y_t + g(\xi_{t+1}),$$

where

$$G(\xi_{t+1}) = \begin{pmatrix} \begin{pmatrix} \xi_{t+1}M(1) & \xi_{t+1}M(1) \\ (1-\xi_{t+1})\,M(2) & (1-\xi_{t+1})\,M(2) \end{pmatrix} & 0_{6\times 6} \\ 0_{6\times 6} & \begin{pmatrix} \xi_{t+1}M(1) & \xi_{t+1}M(1) \\ (1-\xi_{t+1})\,M(2) & (1-\xi_{t+1})\,M(2) \end{pmatrix} \end{pmatrix},$$

$$g(\xi_{t+1})^T = \begin{pmatrix} \xi_{t+1}e_1(1), & 0, & 0, & 0, & 0, & 0, & 0, & 0, & 0, & (1-\xi_{t+1})\,e_1(2), & 0, & 0 \end{pmatrix},$$

$$\xi_{t+1} = \begin{cases} 1 & \text{if } u_{t+1} \in [0, p_{1s_t}], \\ 0 & \text{otherwise,} \end{cases}$$

$$s_{t+1} = \xi_{t+1} + 2\left(1-\xi_{t+1}\right),$$

u_{t+1} is an independently and identically distributed uniform random variable with support $[0,1]$, $e_1(s')$ is the (potentially different across states s') measure of new loans originated at $t+1$, and $0_{6\times 6}$ denotes a 6×6 matrix full of zeros.

B.2 Incurred losses

Incurred losses measured at date t would be those associated with the non-performing loans that are part of the bank portfolio at date t. So the incurred losses reported at t would be given by

$$IL_t = \sum_{z=1,2}\sum_{s=1,2}\lambda(s)x_t(z,s,3),$$

where $\lambda(s)$ is the expected loss given default (LGD) on a non-performing loan conditional on being at state s in date t. This can be more compactly expressed as

$$IL_t = \widehat{b}y_t, \tag{32}$$

where $\widehat{b} = (0,0,\lambda(1),0,0,\lambda(2),0,0,\lambda(1),0,0,\lambda(2))$.

The expected LGDs conditional on each current state s can be found as functions of the previously specified primitives of the model (state-transition probabilities, probabilities of resolution of the loans in subsequent periods, and loss rates $\widetilde{\lambda}(s')$ suffered if resolution happens in each of the possible future states s') by solving the following system of recursive equations:

$$\lambda(s) = \sum_{s'=1,2} p_{s's}\left[\delta_3(s')\widetilde{\lambda}(s') + (1-\delta_3(s'))\lambda(s')\right], \tag{33}$$

for s=1, 2.

B.3 Discounted one-year expected losses

Based on the loan portfolio held by the bank at t, this allowance adds to the incurred losses written above the losses due to default events expected to occur within the immediately incoming year. Since a period in the model is one year, the corresponding allowances are given by:

$$EL_t^{1Y} = (b_\beta + \widehat{b})y_t, \tag{34}$$

where $b_\beta = (\beta_1 b, \beta_2 b)$, $\beta_z = 1/(1+c_z)$, and $b = (b_{11}, b_{12}, 0, b_{21}, b_{22}, 0)$, with

$$b_{sj} = \sum_{s'=1,2} p_{s's}PD_j(s')\left\{[\delta_3(s')/2]\,\widetilde{\lambda}(s') + [1-\delta_3(s')/2]\,\lambda(s')\right\}, \tag{35}$$

for j=1, 2. The coefficients defined in (35) attribute one-year expected losses to loans rated j=1, 2 in state s by taking into account their PDs and LGDs over each of the possible states s' that can be reached at $t+1$, where the corresponding s' are weighted by their probability of occurring given s. The losses associated these one-year ahead defaults are discounted using the contractual interest rate of the loans, c_z, as set at their origination. In Section B.6 we derive an expression for the endogenous value of such rate under our assumptions on loan pricing. As for the loans that are already non-performing (j=3) at date t, the term $\widehat{b}y_t$ in (34) implies attributing to them their conditional-on-s LGD, exactly as in (32).

B.4 Discounted lifetime expected losses

Allowances computed on an lifetime expected basis imply taking into account not just the default events that may affect the currently performing loans in the next year but also those occurring in any subsequent period. Building on prior notation and the same approach explained for the model without aggregate risk, these allowances can be computed as:

$$\begin{aligned} EL_t^{LT} &= b_\beta y_t + b_\beta M_\beta y_t + b_\beta M_\beta^2 y_t + b_\beta M_\beta^3 y_t + ... + \widehat{b} y_t \\ &= b_\beta (I + M_\beta + M_\beta^2 + M_\beta^3 + ...) y_t + \widehat{b} y_t \\ &= b_\beta (I - M_\beta)^{-1} y_t + \widehat{b} y_t = (b_\beta B_\beta + \widehat{b}) y_t, \end{aligned} \tag{36}$$

with

$$M_\beta = \begin{pmatrix} \beta_1 M_p & 0_{6\times6} \\ 0_{6\times6} & \beta_2 M_p \end{pmatrix},$$

$$M_p = \begin{pmatrix} p_{11} M(1) & p_{12} M(1) \\ p_{21} M(2) & p_{22} M(2) \end{pmatrix},$$

$$M(s') = \begin{pmatrix} m_{11}(s') & m_{12}(s') & 0 \\ m_{21}(s') & m_{22}(s') & 0 \\ (1-\delta_3(s')/2) PD_1(s') & (1-\delta_3(s')/2) PD_2(s') & (1-\delta_3(s')) \end{pmatrix},$$

and $m_{ij}(s') = (1-\delta_j(s')) a_{ij}(s')$.

B.5 Discounted expected losses under IFRS 9

As already mentioned, IFRS 9 adopts a hybrid approach that combines the one-year-ahead and life-time approaches described above. Specifically, it applies the one-year-ahead measurement to loans whose credit quality has not significantly increased since origination. For us, these are the loans with j=1, that is, those in the components $x_t(z, s, 1)$ of y_t. In contrast, its applies the life-time measurement to loans whose credit risk has significantly increased since origination. For us, these are the loans with j=2, that is, in the components $x_t(z,s,2)$ of y_t.

As in the case without aggregate risk, it is convenient to split vector y_t into a new auxiliary vector

$$\hat{y}_t = \begin{pmatrix} x_t(1,1,1) \\ 0 \\ 0 \\ x_t(1,2,1) \\ 0 \\ 0 \\ x_t(2,1,1) \\ 0 \\ 0 \\ x_t(2,2,1) \\ 0 \\ 0 \end{pmatrix},$$

which contains the loans with j=1, and the difference

$$\tilde{y}_t = y_t - \hat{y}_t,$$

which contains the rest.

Combining the formulas obtained in (34) and (36), the impairment allowances under IFRS 9 can be compactly described as[30]

$$EL_t^{IFRS9} = b_\beta \hat{y}_t + b_\beta B_\beta \tilde{y}_t + \widehat{b} y_t. \tag{37}$$

B.6 Determining the initial loan rate

Taking advantage of the recursivity of the model, for given values of the contractual interest rates c_z of the loans originated in each of the aggregate states z=1,2, one can obtain the ex-coupon value of a loan originated in state z, when the current aggregate state is s and their current rating is j, $v_j(z,s)$, by solving the system of Bellman-type equations given by:

$$\begin{aligned} v_j(z,s) \;&=\; \mu \sum_{s'=1,2} p_{s's} \Big[(1-PD_j(s'))c_z + (1-PD_j(s'))\delta_j(s') + PD_j(s')(\delta_3(s')/2)(1-\widetilde{\lambda}(s')) \\ &\qquad + m_{1j}(s')v_1(z,s') + m_{2j}(s')v_2(z,s') + m_{3j}(s')v_3(z,s')\Big], \end{aligned} \tag{38}$$

for $(z,s,j) \in \{1,2\} \times \{1,2\} \times \{1,2\}$, and

$$v_j(z,s) = \mu \sum_{s'=1,2} p_{s's}[\delta_3(s')(1-\widetilde{\lambda}(s')) + (1-\delta_3(s'))v_3(z,s')],$$

[30] These definitions clearly imply $EL_t^{IFRS9} = EL_t^{LT} - b_\beta(B_\beta - I)\hat{y}_t \leq EL_t^{LT}$ and $EL_t^{IFRS9} = EL_t^{1Y} + b_\beta(B_\beta - I)\tilde{y}_t \geq EL_t^{1Y}$.

for $(z,s,j) \in \{1,2\} \times \{1,2\} \times \{3\}$.

Under perfect competition and using the fact that all loans are assumed to be of credit quality j=1 at origination, the interest rates c_z can be found as those that make $v(z,z,1)=1$ for z=1,2, respectively.

B.7 Implications for P/L and CET1

By trivially extending the formula derived for the case without aggregate risk, the result of the P/L account with aggregate risk can be written as

$$\begin{aligned} PL_t \;\; = & \sum_{z=1,2} \left\{ \sum_{j=1,2} \left[c_z(1-PD_j(s_t)) - \frac{\delta_3(s_t)}{2} PD_j(s_t)\widetilde{\lambda}(s_t) \right] x_{t\text{-}1}(z,s_t,j) - \delta_3(s_t)\widetilde{\lambda}(s_t) x_{t\text{-}1}(z,s_t,3) \right\} \\ & -r \left(\sum_{z=1,2} \sum_{j=1,2,3} x_{t\text{-}1}(z,s_t,j) - a_{t\text{-}1} - k_{t\text{-}1} \right) - \Delta a_t, \end{aligned} \tag{39}$$

which differs from (20) in the dependence on s_t, the aggregate state at the end of period t, of several of the relevant parameters affecting the default, resolution, and loss upon resolution of the loans.

With the same logic as in the baseline model, dividends and equity injections are now determined by

$$\text{div}_t = \max[(k_{t-1} + PL_t) - 1.3125\underline{k}_t, 0] \tag{40}$$

and

$$\text{recap}_t = \max[\underline{k}_t - (k_{t-1} + PL_t), 0]. \tag{41}$$

Finally, for IRB banks, the minimum capital requirement is now given by:[31]

$$\underline{k}_t^{IRB} = \sum_{j=1,2} \gamma_j(s_t) x_{jt}, \tag{42}$$

and

$$\gamma_j(s_t) = \lambda(s_t) \frac{1 + \left[\left(\sum_{s'} p_{s's_t} \frac{1}{\delta_j(s')} \right) - 2.5 \right] \overline{m}_j}{1 - 1.5\overline{m}_j} \left[\Phi\left(\frac{\Phi^{-1}(\overline{PD}_j) + \text{cor}_j^{0.5}\Phi^{-1}(0.999)}{(1-\text{cor}_j)^{0.5}} \right) - \overline{PD}_j \right], \tag{43}$$

where $\overline{m}_j = [0.11852 - 0.05478 \ln(\overline{PD}_j)]^2$ is a maturity adjustment coefficient, $\Phi(\cdot)$ denotes the cumulative distribution function of a standard normal distribution, $\overline{\text{cor}}_j$ is a correlation coefficient fixed as $\overline{\text{cor}}_j = 0.24 - 0.12(1-\exp(-50\overline{PD}_j))/(1-\exp(-50))$, and

$$\overline{PD}_j = \sum_{i=1,2} \pi_i PD_j(s_i) \tag{44}$$

[31] For SA banks, the equation for the minimum capital requirements in (25) remains valid.

is the through-the-cycle PD for loans rated j (with π_i denoting the unconditional probability of aggregate state i). Equation (44) implies assuming that the bank follows a strict through-the-cycle approach to the calculation of capital requirements (which avoids adding cyclicality to the system through this channel).[32]

[32] Under a point-in-time approach, $\overline{PD}_j$ in (43) should be replaced by $\overline{PD}_j(s_t) = \sum_{s'} p_{s's_t} PD_j(s')$.